

## List of topics

1. Basic facts about matrices: Spectral norm, Frobenius norm, trace, singular value decomposition, orthogonal matrices
2. Random matrices, Gaussian orthogonal ensemble,  $\varrho(H) = \varrho(H_{1,1}, H_{1,2}, \dots, H_{N,N})$
3. Rotational invariance of GOE
4. Wigner's surmise ( $N = 2$ ):  $\delta$ -function notation, classical reformulation
5. Distance function from  $X_1, \dots, X_N$  i.i.d. variables on  $[0, 1]$ , limit for  $N \rightarrow \infty$
6. jpdf  $\varrho(x_1, \dots, x_N)$
7. volumes and areas  $A_{n-1}, V_n$  in  $\mathbb{R}^n$ , “volume”  $\mathbb{V}_n$  and (directly)  $\mathbb{V}_2$
8. Vandermond:  $|J(H \rightarrow \{x, O\})|$  - formulation of the general problem;  $|J(H \rightarrow \{x, O\})|$  for  $N = 2$
9. jpdf  $\varrho(x_1, \dots, x_N)$  from  $\varrho(H_{1,1}, \dots, H_{N,N})$ : statement and proof
10. Vandermond:  $|J(H \rightarrow \{x, O\})|$  - details for general  $N$
11. Exponential of a matrix: definition, basic properties, spectrum, ordering of matrices
12. Trace and “trace is cyclic”, Lie product formula, Golden-Thompson inequality (statement)
13. Scalar Bernstein inequality
14. Non-commutative Bernstein inequality
15. Matrix sparsification - setting, the algorithm, error estimate
16. Matrix multiplication - setting, the algorithm, error estimate
17. Singular value decomposition - setting and the algorithm
18. Matrix completion - setting, statement, basic proof idea