

List of topics:

Essentially, all definitions and theorems from the lecture.

1. Definition of a random process, its paths, finite-dimensional distribution functions, consistent system of X in times (t_1, \dots, t_n) , Daniell-Kolmogorov theorem
2. Mean value, autocovariance function, autocorrelation function of X , positive semi-definite functions, Gauss processes
3. Strongly and weakly stationary processes, Markov processes
4. Wiener process W : definition, $\mu_W(t)$, $C_W(s, t)$, Markov property
5. Lévy's construction of the Wiener process - main idea
6. Time of first passage of the Wiener process, its density
7. Quadratic variance $\langle X \rangle_t$, $\langle W \rangle_t = t$
8. Poisson process N : definition, $\mu_N(t)$, $C_N(s, t)$, Markov property
9. Poisson process: density of the waiting time
10. Limit, continuity and derivative of a random process: definitions, characterizations, examples
11. Definition of the Riemann integral $\int_a^b X_t dt$, characterize the convergence
12. Ergodic theorem for weakly stationary processes
13. Karhunen-Loève expansion
14. Karhunen-Loève for the Wiener process
15. Process with orthogonal increments, distribution function F , $L_2(F)$, definition of $\int_0^t f(s) dW_s$, $\int_0^t W_s dW_s$
16. Chapman-Kolmogorov equations for Markov chains with discrete and continuous time
17. Yule process - problem, main idea of the solution
18. Conditional probability density, Chapman-Kolmogorov equations for Markov processes
19. Kolmogorov differential equations for Markov chains with continuous time
20. Diffusion processes