(M|M|3)

- Příchody: • střední doba 10 s, exp. rozdělení λ
  • 3 pokladny, 20 s μ

Stavy 0, 1, 2, 3, ... počet zákazníků na pokladnách i na frontě

$$Q: \begin{pmatrix} -\lambda & \lambda & 0 & 0 & \dots & & \\ \mu & -\lambda-\mu & \lambda & 0 & \dots & & \\ 0 & 2\mu & -\lambda-2\mu & \lambda & 0 & \dots & \\ 0 & 0 & 3\mu & -\lambda-3\mu & \lambda & 0 & \\ 0 & 0 & 0 & 3\mu; & (-\lambda-3\mu); & \lambda & 0 \dots \end{pmatrix}$$

$\pi Q = 0 \ : \ \pi = [\pi_0 \ \pi_1 \ \pi_2 \ \dots]$

$0 = -\lambda\pi_0 + \mu\pi_1 \ ; \ 0 = \lambda\pi_0 + (-\lambda-\mu)\pi_1 + 2\mu\pi_2 \ ; \ 0 = \lambda\pi_1 - (\lambda+2\mu)\pi_2 + 3\mu\pi_3$

$0 = \lambda\pi_{j-1} - (\lambda+3\mu)\pi_j + 3\mu\pi_{j+1} \quad , j = 3, 4, 5, \dots$

$\pi_1 = \frac{\lambda}{\mu}\pi_0 \ ; \ \pi_2 = \frac{-\lambda\pi_0 + (\lambda+\mu)\pi_1}{2\mu} = \pi_0 \cdot \frac{-\lambda + (\lambda+\mu)\lambda/\mu}{2\mu} = \pi_0 \cdot \frac{-\lambda\mu + \lambda^2 + \lambda\mu}{2\mu^2} = \pi_0 \frac{\lambda^2}{2\mu^2}$

$\pi_3 = \frac{-\lambda\pi_1 + (\lambda+2\mu)\pi_2}{3\mu} = \pi_0 \cdot \frac{-\frac{\lambda^2}{\mu} + (\lambda+2\mu)\frac{\lambda^2}{2\mu^2}}{3\mu} = \frac{\pi_0}{3\mu^3}\left(-\lambda^2\mu + \frac{\lambda^3}{2} + \lambda^2\mu\right) = \frac{\pi_0 \cdot \lambda^3}{6\mu^3}$

$\pi_4 = \frac{-\lambda\pi_2 + (\lambda+3\mu)\pi_3}{3\mu} = \frac{\pi_0}{3\mu}\left(\frac{-\lambda^3}{2\mu^2} + (\lambda+3\mu)\frac{\lambda^3}{6\mu^3}\right) = \frac{\pi_0}{3\mu^4}\left(-\cancel{\frac{\mu\lambda^3}{2}} + (\lambda + \cancel{3\mu})\frac{\lambda^3}{6}\right) = \pi_0 \frac{\lambda^4}{3 \cdot 6 \cdot \mu^4}$

$\dots \pi_j = \pi_0 \cdot \frac{\lambda^j}{3^{j-3} \cdot 6 \cdot \mu^j} \quad \dots \quad \sum_{j=0}^{\infty} \pi_j = 1$ má řešení pro $\lambda < 3\mu$

- π je stacionární stav
- Průměrný počet právě obsluhovaných zákazníků:
  - $\sum_{j=0}^{\infty} j\pi_j$ ... v celém systému "pokladny-fronta"
  - $0\cdot\pi_0 + 1\cdot\pi_1 + 2\pi_2 + 3(1-\pi_0-\pi_1-\pi_2)$ ... na pokladnách
  - $0\cdot(\pi_0+\pi_1+\pi_2+\pi_3) + \pi_4 + 2\pi_5 + \dots$ ... ve frontě
- Pravd., že zákazník nebude muset čekat ve frontě: $\pi_0+\pi_1+\pi_2$.

$(M|M|\infty)$: λ, μ

$$Q: \begin{pmatrix} -\lambda & \lambda & 0 & \dots & & \\ \mu & -(\lambda+\mu) & \lambda & 0 & \dots & \\ 0 & 2\mu & -(\lambda+2\mu) & \lambda & 0 & \\ & & & & \ddots & \end{pmatrix}; \quad \pi Q = 0:$$

$$\begin{aligned} -\lambda\pi_0 + \mu\pi_1 &= 0 \\ \lambda\pi_0 - (\lambda+\mu)\pi_1 + 2\mu\pi_2 &= 0 \\ \lambda\pi_1 - (\lambda+2\mu)\pi_2 + 3\mu\pi_3 &= 0 \\ &\vdots \\ \lambda\pi_{j-1} - (\lambda+j\mu)\pi_j + (j+1)\mu\pi_{j+1} &= 0 \\ &\vdots \\ j &\geq 1 \end{aligned}$$

$$\pi_1 = \frac{\lambda}{\mu}\pi_0; \quad \pi_2 = \frac{-\lambda\pi_0 + (\lambda+\mu)\pi_1}{2\mu} = \pi_0 \cdot \frac{-\lambda + (\lambda+\mu)\lambda/\mu}{2\mu} =$$

$$= \pi_0 \cdot \frac{-\lambda\mu + \lambda^2 + \lambda\mu}{2\mu^2} = \pi_0 \cdot \frac{\lambda^2}{2\mu^2}; \quad \dots \pi_j = \pi_0 \cdot \frac{\lambda^j}{j!\,\mu^j}, \; j \geq 0$$

Opravdu: $$\pi_{j+1} = \frac{1}{(j+1)\mu}\left\{-\lambda\pi_0 \frac{\lambda^{j-1}}{(j-1)!\,\mu^{j-1}} + (\lambda+j\mu)\pi_0 \frac{\lambda^j}{j!\,\mu^j}\right\}$$

$$= \pi_0\left\{\cancel{\frac{-\lambda^j}{(j+1)(j-1)!\,\mu^j}} + \frac{\lambda^{j+1}}{(j+1)!\,\mu^{j+1}} + \cancel{\frac{\lambda^j}{(j+1)(j-1)!\,\mu^j}}\right\} = \pi_0 \cdot \frac{\lambda^{j+1}}{(j+1)!\,\mu^{j+1}}$$

- Konvergují vždy! ... $\boxed{\pi_k}$