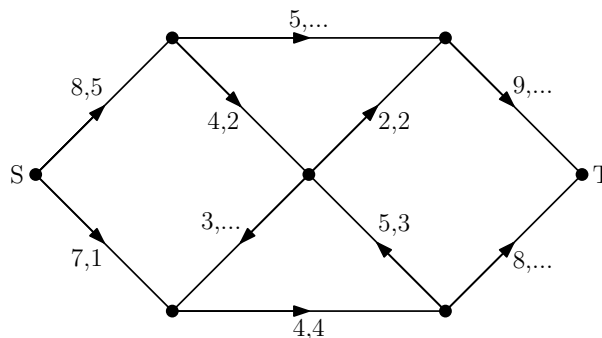# Graph Theory — sample questions from the final exam

1. In the network below, the first number on each edge is the capacity of the edge. The second number is the flow value on the edge for an initial flow.

   a) Find the missing flow values.

   b) Use this initial flow to find a maximal flow and minimal cut (S = source/producer, T = target/consumer)


2. Decide whether the following propositions are true or false. If a proposition is true give reasoning behind your answer, if it is false provide a counterexample.

   a) Any subgraph (with at least two vertices) of a bipartite graph is also bipartite.

   b) Any subgraph of a complete graph is also complete.

   c) Any hamiltonian graph is connected.

3. Is there an eulerian closed walk in the following graph? If there is give a corresponding sequence of vertices, if there is not find a minimal possible addition (minimal in the number of added edges) to this graph which will guarantee the presence of an eulerian closed walk and then give a corresponding sequence of vertices.
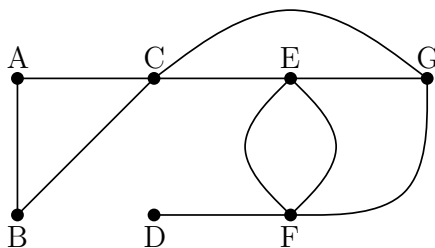
BONUS Question: Does such simple addition always exist? That is, is it true that for any graph $G = (V, E)$ there exists a finite set of edges E' such that the graph $G' = (V, E \cup E')$ is eulerian? Give reasoning behind your answer.

4. Is the graph drawn below planar? If it is find its planar drawing, if it is not explain why not.
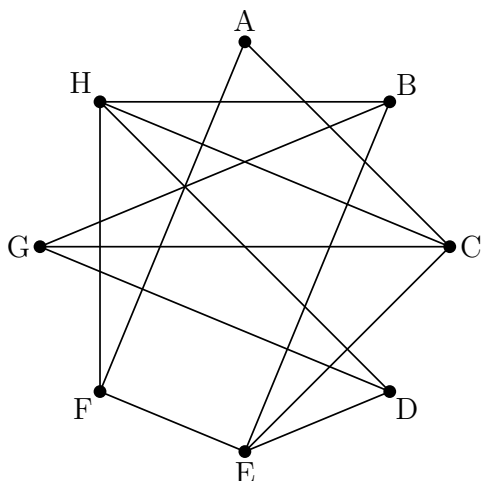
5. Determine if the following sequences are graphic, i.e. if they correspond to sequences of degrees of a graph. If a sequence is graphic, give a graph that realizes it, otherwise explain why it is not.

a) 6,4,4,4,4,3,3

b) 5,4,3,2,2