

Read the following numbers:

25; 19; 9; 2,279; 103; 1,000,000; 14; 40; 104; 138; 500; 44,005; 2,004; 836; 1,017; 6,000; 82,985; 10,000,000; 200,000; 15; 50; 629; 2,102; 12; 17; 70; 708; 7,008; 1,825; 1,901; 348; 990; 3,000; 8,000,000; 1,621; 3,508; 3,528; 3,500; 180; 18; 6,213; 963; 2,000,000,000; 1,526;

1. Read :

- a) $2 + 5 =$; $10 + 8 =$; $25 + 15 =$; $78 + 7 =$; $49 + 9 =$; $99 + 1 =$;
 $129 + 37 =$; $371 + 371 =$; $a + b$, $-x + 1$, $x + y$, $1 + y$
- b) $19 - 7 =$; $23 - 9 =$; $91 - 18 =$; $11 - 3 =$; $20 - 10 =$; $150 - 100 =$;
 $1050 - 85 =$; $5,000 - 3,000 =$; $a - x$, $x - 1$, $a - b$
- c) $5.5 =$, $3.8 =$, $7.7 =$, $4.12 =$, $13.10 =$, $6.9 =$, $ab = c$, $xy = z$, $2ab$
- d) $9:3 =$, $5:1 =$, $21:7 =$, $27:9 =$, $35:5 =$, $100:10 =$, $48:12 =$, $75:15 =$,
 $a:b = x$, $X:Z = y$
- e) $2/3$, $4/7$, $1/2$, $3/4$, $1/10$, $5/100$, $3/1000$, $6/21$, $4/3$, $5/2$
 a/b , b^2/c , α/y , $\pi/2$, $1/x$, $x/2$, $1/\sqrt{x}$, $c+d/c-d$
- f) 0.1 ; $.1$; $.002$; 0.003 ; $.2334$; 5.1 ; 7.99 ; 10.5 ; 100.25 ;
- g) 2^2 , 2^3 , a^2 , a^{-2} , a^3 , a^{-3} , $(x^2 + y^2) = z$, $a^2 + b^2$, $(a + b)^3$, $(a + b)^m$,
 $a^n a^m$, $a^m \cdot a^n = a^{m+n}$, $1/a^n = a^{-n}$, $a^m/a^n = a^{m-n}$, $(a^m)^n = a^{mn}$, $(a/b)^m = a^m/b^m$,
 $(a+b)^{-1}$, x^{-1} , x^{-1} , $a^{1/3}$, $a^{-1/3}$, a^x , $a^{1/x}$, $a^{-1/x}$, $(a^{2/3})^x$
- h) \sqrt{x} , $\sqrt[3]{a}$, $\sqrt[4]{x+1}$, $\sqrt[n]{y}$, $\sqrt{-2a}$, $\sqrt{-3x}$, $\sqrt[n]{a^n} \cdot a$, $\sqrt[n]{a} = a^{1/n}$,
 $\sqrt[m]{a^n} = (\sqrt[m]{a})^n = a^{n/m}$, $\sqrt[n]{1/a} = 1/\sqrt[n]{a} = a^{-1/n}$,
 $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$, $a\sqrt[n]{b} = \sqrt[n]{a^n b}$, $\sqrt[n]{0} = 0$

2. Read the following signs :

0 ; $+$; $-$; \pm ; $a \cdot b$; $a:b$; $x:y = a:b$; $=$; \equiv ; \neq ; \approx ; \doteq ; $a > b$; $b < a$;
 $a \nless b$; $y \less x$; $()$; $[]$; \tilde{a} ; a^* ; \bar{a} ; \bar{a} ; a^{**} ; a' ; a'' ; a_n ; x_1 ; y_2 ;
 $|a|$; $n!$; \rightarrow ; \Rightarrow ; X ; a ; \int ; \iint ; ∞

3. Read the following letters of Greek alphabet, giving their Czech names :

α , β , γ , δ , ω , Δ , δ , ν , λ , φ , μ , ν , ρ , η , ϵ , τ , χ , ψ ,
 α , ξ , ξ , υ , \omicron , ν ; Σ , Π , Ω

4. Pay attention to stress :

difference, different, to differentiate, differentiation, differential
add, addition, additional
subtract, subtraction
multiply, multiplication
divide, division
integrate, integral, integration

5. Translate into Czech :

- a) equation, expression, formula, theorem, theory, quantity, constant, value, property, relation, variable, to define, definition
- b) accordingly, hence, thence, whence
- c) let us assume that ... ; let $a = b$; let P denote ; let Equation 1 denote ... ; according to Eq.2, let $a = b$;
- d) the equation is valid for ... ; (b) is true, if ; Eq.5 holds for ... ; the relation applies for all values of ... ;
- e) Eq.1 may be expressed as ... ; we can express Eq.1 as ... ; Eq.3 may be written in the form ; P can be written as ... ; the following equation may be put as

6. Translate into English :

rovnice, sčítání, odčítání, násobení, dělení, výsledek, mocnění, mocnina, index, odmocňování, kořen, zlomek, v čitateli, ve jmenovateli, lomeno, funkce, matematická analýza, derivování, derivovat, odvodit, diferenciál, integrál, integrování, sčítat, odčítat, násobeno 5, děleno 5, umocnit na druhou, umocnit na třetí, mocnit, derivovat, integrovat, odmocňovat

Case Study: Newton's Law of Cooling

When a hot liquid is placed in a cooler environment, _____ observation shows that its temperature decreases to _____ that of its surroundings. A typical graph of the temperature of the liquid _____ against time is shown in Figure 1

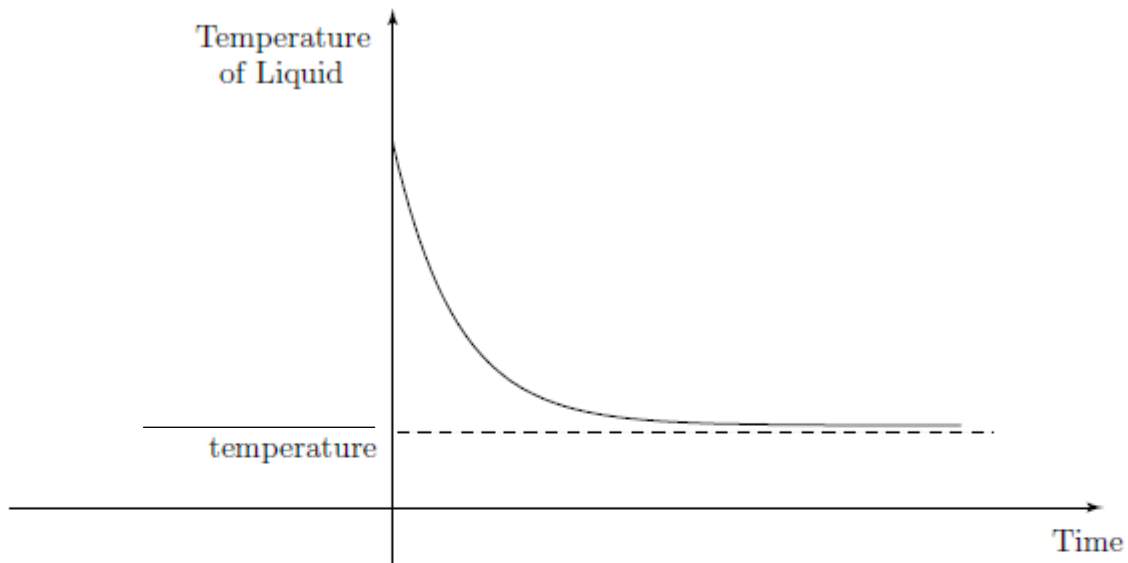


Figure 1:

After an _____ rapid _____ the temperature changes progressively _____ rapidly and _____ the curve appears to 'flatten out'.

Newton's law of cooling states that the rate of cooling of liquid is _____ to the difference between its temperature and the temperature of its _____.

To _____ this into mathematics, let t be the time _____ (in seconds, s), θ the temperature of the liquid ($^{\circ}\text{C}$), and θ_0 the temperature of the liquid at the start ($t = 0$). The temperature of the _____ is _____ by θ_s .

Try each part of this exercise

Write down the mathematical _____ which is _____ to Newton's law and state the accompanying condition.

Part (a) First, find expressions for the _____ of cooling and the difference between the liquid's temperature and that of the environment.

Part (b) Now formulate Newton's law of cooling.

In the _____ example we call t the _____ variable and θ the _____ variable. Since the condition is given at $t = 0$ we refer to it as an _____ condition. For reference, the solution of the differential equation which _____ the initial condition is $\theta = \theta_s + (\theta_0 - \theta_s)e^{-kt}$.

Read the text and fill in the missing words:

surroundings	eventually	convert	experimental	plotted
satisfies	independent	initial	initially	above
denoted	rate	equivalent	environment	
surrounding	less	approximately	decrease	
proportional	elapsed	dependent	equation	

The General Solution of a Differential Equation

Consider the formula $y = Ae^{2x}$ where A is an arbitrary constant. If we differentiate it we obtain

$$\frac{dy}{dx} = 2Ae^{2x}$$

and so, since $y = Ae^{2x}$ we obtain

$$\frac{dy}{dx} = 2y.$$

Then the differential equation satisfied by y is

$$\frac{dy}{dx} = 2y.$$

Notice that we have eliminated the arbitrary constant.

Now consider the formula

$$y = A \cos 3x + B \sin 3x$$

where A and B are arbitrary constants. Differentiating, we obtain

$$\frac{dy}{dx} = -3A \sin 3x + 3B \cos 3x.$$

Differentiating a second time gives

$$\frac{d^2y}{dx^2} = -9A \cos 3x - 9B \sin 3x.$$

The right-hand side is simply (-9) times the expression for y . Hence y satisfies the differential equation

$$\frac{d^2y}{dx^2} = -9y.$$

Now do this exercise

Find a differential equation satisfied by $y = A \cosh 2x + B \sinh 2x$ where A and B are arbitrary constants.

We have seen that an expression including one arbitrary constant required one differentiation to obtain a differential equation which eliminated the arbitrary constant. Where two constants were present, two differentiations were required. Is the converse true? For example, would a differential equation involving $\frac{dy}{dx}$ as the only derivative have a general solution with one arbitrary constant and would a differential equation which had $\frac{d^2y}{dx^2}$ as the highest derivative produce a general solution with two arbitrary constants? The answer is, usually, yes.