Rényi transfer entropy and its application for intraday financial data

Petr Jizba[1,2], Jan Korbel[1,3] and Mohammad Shefaat[2]

[1]Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University in Prague, Břehová 7, 11519, Prague
[2]Institut für Theoretische Physik, Freie Universität Berlin, Arnimallee 14, 14195, Berlin
[3]Max-Planck-Institut für Wissenschaftsgeschichte, Boltzmannstraße 22, 14195, Berlin

Abstract

Measuring information transfer between time series is a challenging task. Classical statistical approaches based on correlations do not provide complete image about sources of the information flow. On the other hand, there have been introduced many sophisticated approaches that enable us to reveal the complex nature of many processes. One of these successful approaches is based on transfer entropy, introduced by Schreiber [1] and generalized to the class of q-Rényi transfer entropies by Jizba et al. [2]. The latter enables to 'zoom' different part of distribution. The whole concept of transfer entropy enables to reveal not only strength of information flow, but also the direction. This method is applied to daily and intraday financial data to observe information flows among exchanges. The method shows the whole complexity of informational flows among particular stocks, which may also differ on different timescales.

Entropy as an informational measure

- Shannon entropy (SE) is a classical concept from information theory that enables us to measure the amount of information encoded in the data. It is defined as:

$$H(X) = -\sum_{x \in X} p(x) \log p(x)\,. \tag{1}$$

- In case of two random variables X and Y, we can define the conditional entropy, i.e. the average information entropy gain by X, when given Y

$$H(X|Y) = \sum_{y \in Y} p(y) H(X|Y=y)$$
$$= -\sum_{x \in X, y \in Y} p_{X,Y}(x,y) \log p_{X|Y}(x|y)\,. \tag{2}$$

- Further, one can define mutual information (SMI) $I(X,Y)$ as

$$I(X,Y) = H(X) - H(X|Y) = H(Y) - H(Y|X)$$
$$= \sum_{x \in X, y \in Y} p_{X,Y}(x,y) \log \frac{p_{X,Y}(x,y)}{p_X(x), p_Y(y)}\,. \tag{3}$$

- Mutual information is unfortunately not directional and does not obey causality, therefore Schreiber [1] introduced the informational measure called Transfer entropy on discrete stochastic processes $\{X_i\}_{i=1}^{\infty}$, $\{Y_j\}_{j=1}^{\infty}$

$$\begin{aligned} T_{Y\to X}(m,l) &= H(X_{m+1}|X_m,\dots,X_1) \\ &\quad - H(X_{m+1}|X_m,\dots,X_1,Y_m,\dots,Y_{m-l+1}) \\ &= I(X_{m+1},X_m,\dots,X_1,Y_m,\dots,Y_{m-l+1}) \\ &\quad - I(X_{m+1},X_m,\dots,X_1). \end{aligned} \tag{4}$$

- The history can be truncated only for Markovian systems. For Non-Markovian systems, one should take the whole history up to $t \to -\infty$. This is not possible in case of discrete finite data, therefore Marchinski et al. [3] introduced Effective transfer entropy (ETE)

$$T^{eff}_{Y\to X}(m,l) = T_{Y\to X}(m,l) - T_{Y_{shuffled}\to X}(m,l) \tag{5}$$

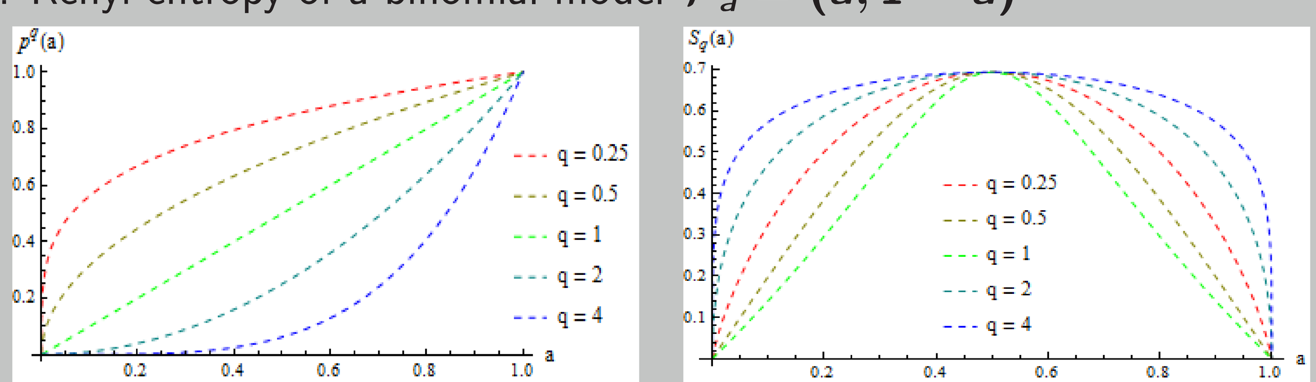
- Because of the finite length of time series, one has to transform the real-numbered time series into the finite alphabet $\{I_1,\dots,I_N\}$, where the probability distribution of this series corresponds to discrete histogram that approximates underlying distribution. The length N is determined from the particular setup of the measured system.

Rényi entropy

- Shannon entropy can be generalized into 1-parameter class of Rényi entropies (RE)

$$S_q(X) = \frac{1}{q-1} \log \sum_{x \in X} p^q(x)\,. \tag{6}$$

- The entropy is motivated by Campbell coding theorem: it minimizes average cost of codeword in case when cost is an exponential function of length.
- The RE accentuates extreme events for $q > 1$ and suppresses them for $q < 1$. For $q \to 1$ we recover SE.
- Example: Rényi entropy of a binomial model $\mathcal{P}_a = (a, 1-a)$



- Analogously to (SMI) one can define Rényi mutual information (RMI) as

$$I_q(X,Y) = \frac{1}{q-1} \log \frac{\sum_{x\in X, y\in Y} p_X^q(x) p_Y^q(y)}{\sum_{x\in X, y\in Y} p_{X,Y}^q(x,y)} \tag{7}$$

- Rényi transfer entropy (RTE) can be straightforwardly defined as

$$\begin{aligned} T_{q;Y\to X}(m,l) &= I_q(X_{m+1},X_m,\dots,X_1,Y_m,\dots,Y_{m-l+1}) \\ &\quad - I_q(X_{m+1},X_m,\dots,X_1) \end{aligned} \tag{8}$$

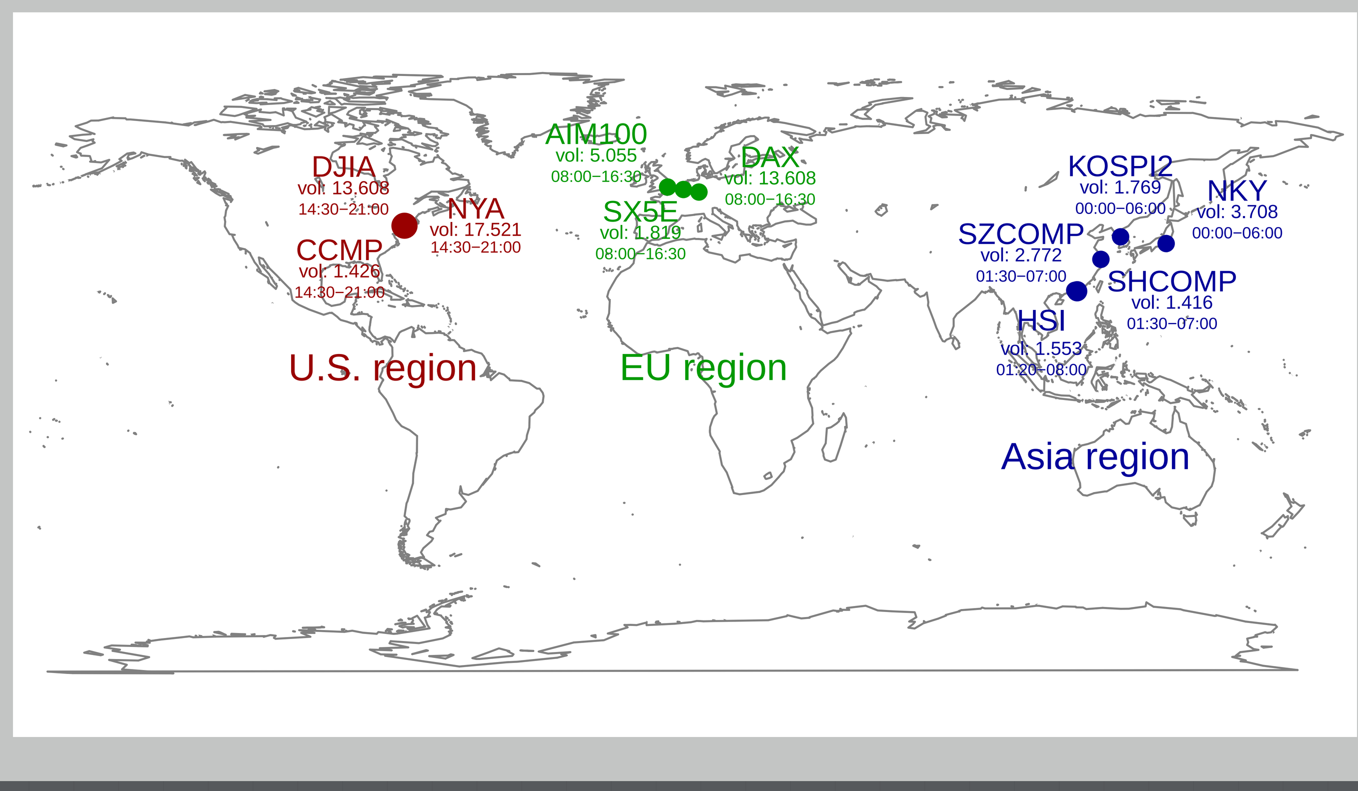
and the effective Rnyi transfer entropy (ERTE) analogously as

$$T^{eff}_{q;Y\to X}(m,l) = T_{q;Y\to X}(m,l) - T_{q;Y_{shuffled}\to X}(m,l). \tag{9}$$

- Contrary to Shannon ETE, which is always positive [2], ERTE can be generally also negative in particular cases, which means that knowledge of Y reveals an extra risk for swan-like events.

Major world stock indices

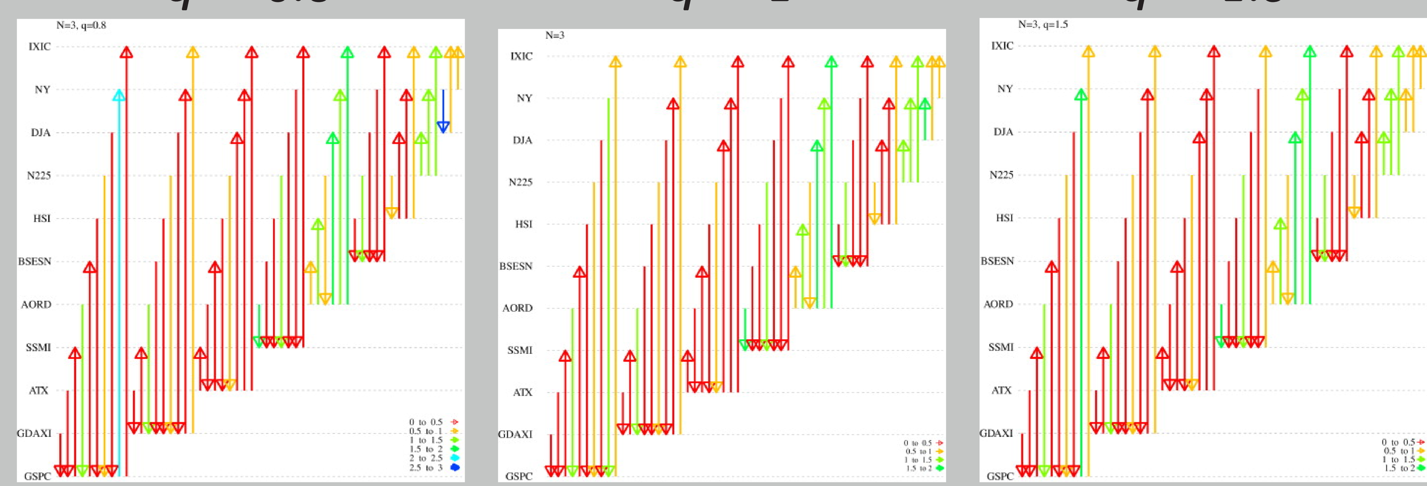
- We take into the account 11 major stock indices according to the traded volume. 3 are from U.S. region, 3 from Europe and 5 from Asia.
- The data are collected on the minute basis (high-frequency), and are collected over the period of 3 years during the crisis 2007-2009.
- We also create the series **Superindex**, which is weighted sum of all indices and should contain all information during the whole day.
- Main properties of the indices:



Net informational flow of daily financial data

- Jizba et al. [2] presented the net entropy flow $F_{X,Y} = T_{X\to Y} - T_{Y\to X}$ among various stocks. One can distinctly observe the different net flows for different values of q. This is caused by accentuating different parts of distribution.

$q = 0.8$ $\quad$ $q = 1$ $\quad$ $q = 1.5$



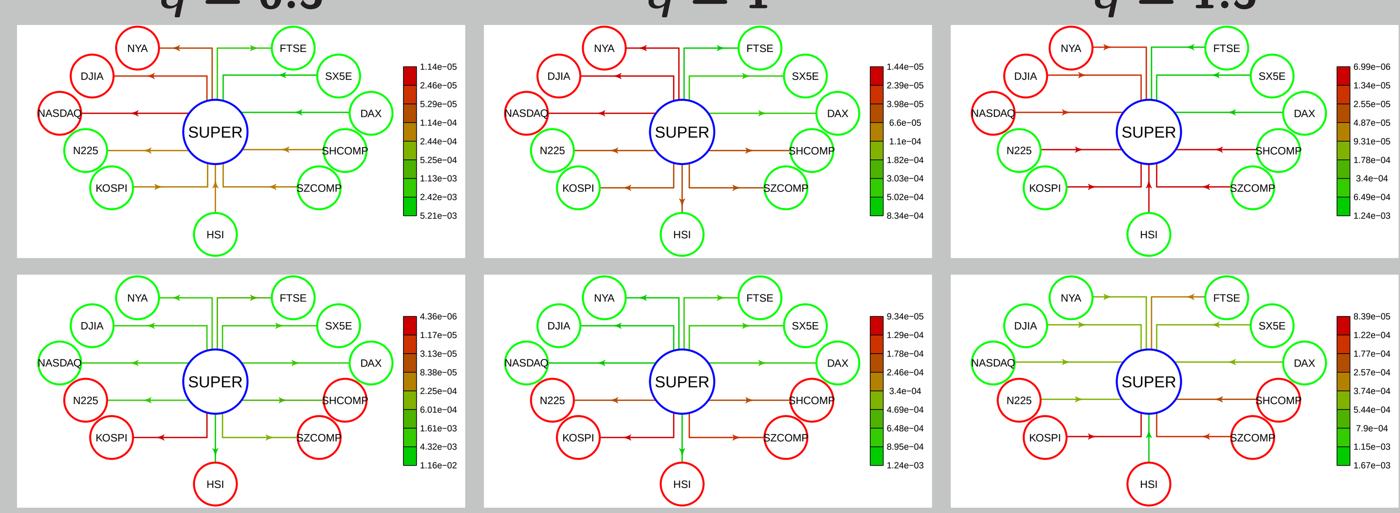
Transfer information from Superindex to particular stocks

- The data are divided into 3-hrs time windows and for each is calculated ETE from **Superindex** to particular stocks. Particularly interesting are time windows when opening times of EU and Asian stocks, resp. EU and U.S. stocks are overlapped. For $q \neq 1$, the opposite direction of arrow means the negative effective information flow.

$q = 0.5$ $\quad$ $q = 1$ $\quad$ $q = 1.5$

6-9h UTC

15-18h UTC



Conclusion

- Transfer entropy is a powerful tool to reveal strength and direction of information flow.
- Informational flow of extreme events, such as sudden jumps, can be revealed by Rényi entropy with appropriate parameter q.
- Financial data exhibit a variety of complex informational flows among different stocks.

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email: korbeja2@fjfi.cvut.cz