Vzorce pro mocniny a odmocniny:

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

$$a^r a^s = a^{r+s}$$

$$\frac{a^r}{a^s} = a^{r-s}$$

$$(a^r)^s = a^{rs}$$

$$(ab)^r = a^r b^r$$

$$\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$$

$$\sqrt[n]{a}\,\sqrt[n]{b} = \sqrt[n]{ab}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$

Pascalův trojúhelník

$$\begin{array}{ccccccccccc} & & & & & 1 & & & & & \\ & & & & 1 & & 1 & & & & \\ & & & 1 & & 2 & & 1 & & & \\ & & 1 & & 3 & & 3 & & 1 & & \\ & 1 & & 4 & & 6 & & 4 & & 1 & \\ 1 & & 5 & & 10 & & 10 & & 5 & & 1 \\ \ddots & & \ddots & & \ddots & & \ddots & & \ddots & & \end{array}$$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Logaritmy

$$\log_a c = b \Leftrightarrow a^b = c$$

$$\log_a a = 1$$

$$\log_a 1 = 0$$

$$a^{\log_x a} = a$$

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

$$log_a x^r = r \log_a x$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Goniometrické vzorce

$$\sin^2 x + \cos^2 x = 1$$

$$\sin x = \pm\sqrt{1 - \cos^2 x}$$

$$\cos x = \pm\sqrt{1 - \sin^2 x}$$

$$\operatorname{cotg} x = \frac{1}{\operatorname{tg} x}$$

$$1 + \operatorname{tg}^2 x = \frac{1}{\cos^2 x}$$

$$1 + \operatorname{cotg}^2 x = \frac{1}{\sin^2 x}$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\sin \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}$$

$$\cos \frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}}$$

$$\operatorname{tg} \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\sin x - \sin y = 2 \sin \frac{x-y}{2} \cos \frac{x+y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\cos x - \cos y = -2 \sin \frac{x-y}{2} \sin \frac{x+y}{2}$$

Hyperbolické vzorce

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

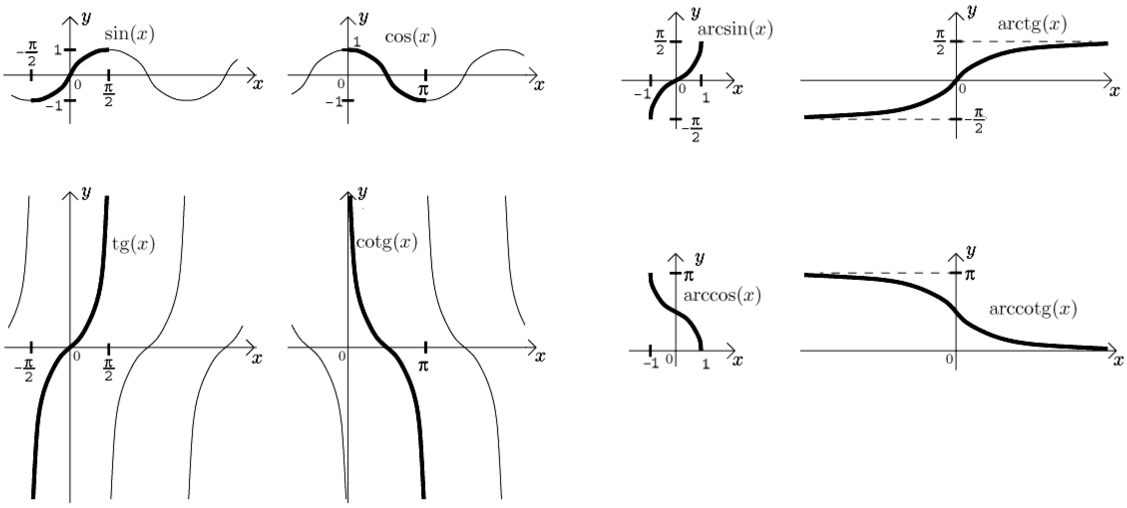
$$\cosh^2 x - \sinh^2 x = 1$$

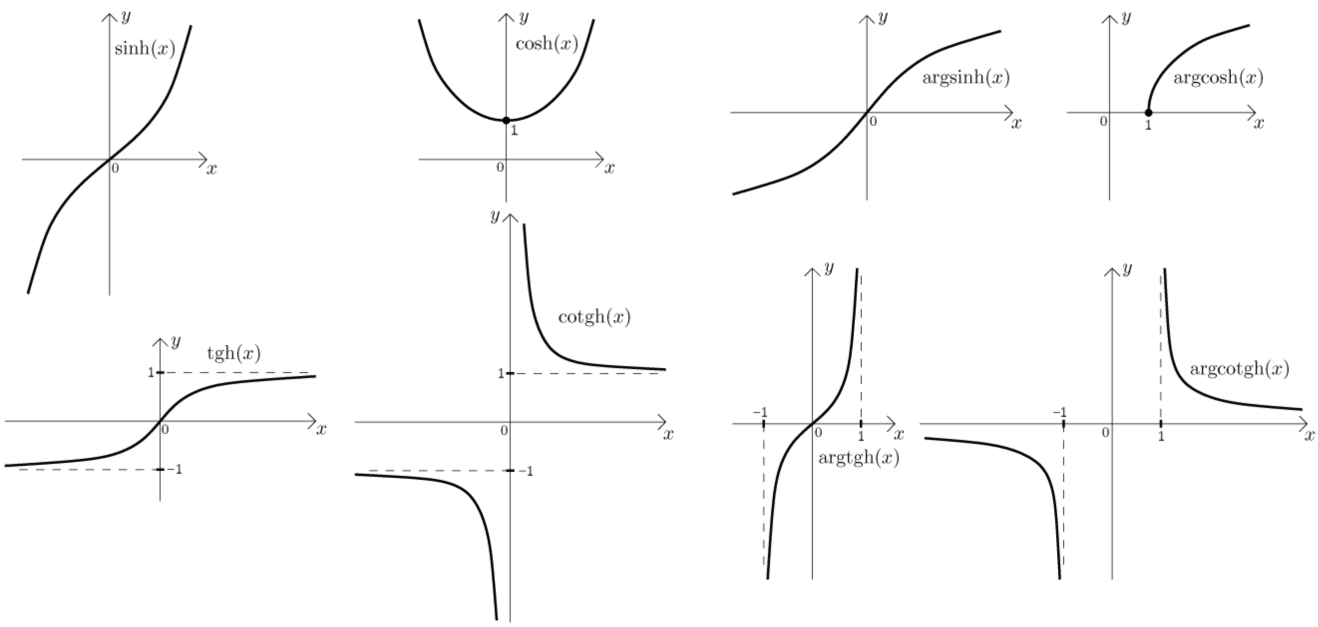
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

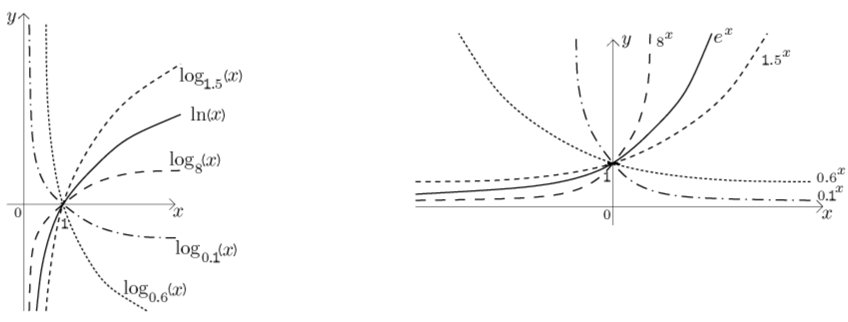
$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh^2 x = \frac{\cosh 2x + 1}{2}$$

$$\sinh^2 x = \frac{\cosh 2x - 1}{2}$$


sin(x)
cos(x)
arcsin(x)
arctg(x)
tg(x)
cotg(x)
arccos(x)
arccotg(x)


sinh(x)
cosh(x)
argsinh(x)
argcosh(x)
tgh(x)
cotgh(x)
argtgh(x)
argcotgh(x)


$\log_{1.5}(x)$
ln(x)
$\log_8(x)$
$\log_{0.1}(x)$
$\log_{0.6}(x)$
8^x
e^x
1.5^x
0.6^x
0.1^x

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x$	0	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	0	-1	0	1
$\operatorname{tg} x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	–	0	–	0
$\operatorname{cotg} x$	–	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	–	0	–

Derivace

$$f'(x) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$(af + bg)' = af' + bg'$$

$$(fg)' = f'g + fg'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$(h(f(x)))' = h'(f(x))f'(x)$$

Derivace funkcí

Funkce	**Derivace**	D_f	$D_{f'}$
$(x+a)^n$	$n(x+a)^{n-1}$	$\mathbb{R}(\mathbb{R} - \{a\}$ pro $n < 0)$	$\mathbb{R}(\mathbb{R} - \{a\}$ pro $n < 0), a \in \mathbb{C}, n \in \mathbb{Z}$
x^α	$\alpha x^{\alpha-1}$	$\mathbb{R}^+$	$\mathbb{R}^+, \alpha \in \mathbb{R}$
$\exp^{ax}$	$a \exp^{ax}$	$\mathbb{R}$	$\mathbb{R}, a \in \mathbb{C}$
$\ln x$	$\frac{1}{x}$	$\mathbb{R}^+$	$\mathbb{R}^+$
a^x	$a^x \ln a$	$\mathbb{R}$	$\mathbb{R}, a > 0$
$\log_a x$	$\frac{1}{x \ln a}$	$\mathbb{R}^+$	$\mathbb{R}^+, a \in (0,1) \cup (1, \infty)$
$\sin x$	$\cos x$	$\mathbb{R}$	$\mathbb{R}$
$\cos x$	$-\sin x$	$\mathbb{R}$	$\mathbb{R}$
$\operatorname{tg} x$	$\frac{1}{\cos^2 x}$	$((2k-1)\frac{\pi}{2}, (2k+1)\frac{\pi}{2}), k \in \mathbb{Z}$	$((2k-1)\frac{\pi}{2}, (2k+1)\frac{\pi}{2}), k \in \mathbb{Z}$
$\operatorname{cotg} x$	$\frac{-1}{\sin^2 x}$	$(k\pi, (k+1)\pi), k \in \mathbb{Z}$	$(k\pi, (k+1)\pi), k \in \mathbb{Z}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$	$[-1, 1]$	$(-1, 1)$
$\arccos x$	$\frac{-1}{\sqrt{1-x^2}}$	$[-1, 1]$	$(-1, 1)$
$\operatorname{arctg} x$	$\frac{1}{1+x^2}$	$\mathbb{R}$	$\mathbb{R}$

Funkce	Derivace	D_f	$D_{f'}$
$\operatorname{arccotg} x$	$\frac{-1}{1+x^2}$	$\mathbb{R}$	$\mathbb{R}$
$\sinh x$	$\cosh x$	$\mathbb{R}$	$\mathbb{R}$
$\cosh x$	$\sinh x$	$\mathbb{R}$	$\mathbb{R}$
$\operatorname{tgh} x$	$\frac{1}{\cosh^2 x}$	$\mathbb{R}$	$\mathbb{R}$
$\operatorname{cotgh} x$	$\frac{-1}{\sinh^2 x}$	$\mathbb{R} - \{0\}$	$\mathbb{R} - \{0\}$
$\operatorname{argsinh} x$	$\frac{1}{\sqrt{1+x^2}}$	$\mathbb{R}$	$\mathbb{R}$
$\operatorname{argcosh} x$	$\frac{1}{\sqrt{x^2-1}}$	$[1, \infty)$	$(1, \infty)$
$\operatorname{argtgh} x$	$\frac{1}{1-x^2}$	$(-1, 1)$	$(-1, 1)$
$\operatorname{argcotgh} x$	$\frac{1}{1-x^2}$	$\mathbb{R} - [-1, 1]$	$\mathbb{R} - [-1, 1]$

Primitivní funkce

$$\int (f + \alpha g) = F + \alpha G \text{ na } M$$

Per partes: $\int f'g = fg - \int fg'$

1. substituční metoda: $\int (f \circ \varphi)\varphi' = F \circ \varphi$

2. substituční metoda: $\int (f \circ \varphi)\varphi' = G \Rightarrow \int f = G \circ \varphi^{-1}$

$$\int_a^b f = -\int_b^a f \qquad \int_a^a f = 0$$

Zobecněná Newtonova formule $\int_a^b f = [F]_a^b = F(b) - F(a)$

1. věta o střední hodnotě: $f(x) \geq 0 \forall x \in \langle a, b \rangle : \int_a^b fg = \mu \int_a^b f, \mu \in \left\langle \inf g_{\langle a,b \rangle}, \sup g_{\langle a,b \rangle} \right\rangle$

2. věta o střední hodnotě: g monotónní na $\langle a, b \rangle : \exists \xi \in \langle a, b \rangle : \int_a^b fg = g(a) \int_a^\xi f + g(b) \int_\xi^b f$

Zobecněný Riemannův integrál - Newtonova formule: $\int_a^b f = \lim_{b-} F - \lim_{a+} F$

Substituce: $y = \operatorname{tg} x, \sin^2 x = \frac{y^2}{y^2+1}, \cos^2 x = \frac{1}{y^2+1}, dx = \frac{dy}{y^2+1}$

$$y = \operatorname{tg} \frac{x}{2}, \sin x = \frac{2y}{y^2+1}, \cos x = \frac{1-y^2}{y^2+1}, \operatorname{tg} x = \frac{2y}{1-y^2}, dx = \frac{2}{y^2+1} dy,$$

$$\sin^2 \frac{x}{2} = \frac{y^2}{1+y^2}, \cos^2 \frac{x}{2} = \frac{1}{1+y^2}$$

Eulerovy substituce: $\sqrt{ax^2+bx+c}, x \in \mathbb{R}$

$$ax^2+bx+c = a(x-x_1)(x-x_2), x_1 < x_2, t = \frac{x-x_1}{x-x_2}$$

$$a > 0 : \sqrt{ax^2+bx+c} = \sqrt{a}x + t$$

$$c > 0 : \sqrt{ax^2+bx+c} = xt + \sqrt{c}$$

$$\int R(e^x) = \int \frac{e^x R(e^x)}{e^x} = \left\{e^x = t \qquad e^x dx = dt\right\} = \int \frac{R(t)}{t}$$

$$\int \frac{R(\ln x)}{x} = \left\{\ln x = t \qquad \frac{dx}{x} = dt\right\} = \int R(t)dt$$

Nelimitní tvar srovnávacího kritéria: $\exists \int_a^x f, \int_a^x g \forall x \in (a,b), 0 \leq f(x) \leq g(x) :$

$$\int_a^b g \quad K \Rightarrow \int_a^b f \quad K \qquad \int_a^b f \quad D \Rightarrow \int_a^b g \quad D$$

Limitní tvar srovnávacího kritéria: $\exists \int_a^x f, \int_a^x g \forall x \in (a,b), 0 \leq f(x), 0 \leq g(x) : \lim_{b-} \frac{f}{g} = d$

$$d \in \mathbb{R} \int_a^b g \quad K \Rightarrow \int_a^b f \quad K \qquad d \neq 0 \int_a^b g \quad D \Rightarrow \int_a^b f \quad D$$

Dirichlet. krit.: $\exists F(x) = \int_a^x f \forall x \in (a,b)$ omezená na , g monotónní na $\langle a,b)$, $\lim_{b-} g = 0 \Rightarrow \int_a^b fg \quad K$

Abel. krit.: $\exists \int_a^x f \forall x \in (a,b)$ konverguje, g monotónní, omezená na $\langle a,b) \Rightarrow \int_a^b fg \quad K$

$$\int_{-e}^{+\infty} \frac{dx}{x \ln^\alpha x} = \left\{\ln x = t \qquad \frac{dx}{x} = dt\right\} = \int_1^{+\infty} \frac{dt}{t^\alpha} \text{ pro } \alpha > 1 \text{ konverguje } = \frac{1}{\alpha - 1}$$

Primitivní funkce	**Definiční obor**
$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1} + C$	$\mathbb{R} - \{-a\}, n < 0$, pozn. $n \neq -1 . n \in \mathbb{Z}, a \in \mathbb{C}$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C$	$\mathbb{R}^+$ pozn. $\alpha \in \mathbb{R} - \{-1\}$
$\int \frac{1}{x+a} dx = \ln\|x+a\| + C$	$\mathbb{R} - \{-a\}$ pozn. $a \in \mathbb{R}$
$\int \exp^{ax} dx = \frac{1}{a} \exp^{ax} + C$	$\mathbb{R}$ pozn. $a \in \mathbb{C}, a \neq 0$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\mathbb{R}$ pozn. $a > 0, a \neq 1$
$\int \cos x dx = \sin x + C$	$\mathbb{R}$
$\int \sin x dx = -\cos x + C$	$\mathbb{R}$
$\int \operatorname{tg} x dx = -\ln\|\cos x\| + C$	$x \in \mathbb{R} - \{\frac{\pi}{2} + k\pi, k \in \mathbb{Z}\}$
$\int \operatorname{cotg} x dx = \ln\|\sin x\| + C$	$x \in \mathbb{R} - \{k\pi, k \in \mathbb{Z}\}$

Primitivní funkce	**Definiční obor**
$\int \frac{1}{\cos^2 x} dx = \operatorname{tg} x + C$	$((2k-1)\frac{\pi}{2}, (2k+1)\frac{\pi}{2}), k \in \mathbb{Z}$
$\int \frac{1}{\sin^2 x} dx = -\operatorname{cotg} x + C$	$(k\pi, (k+1)\pi), k \in \mathbb{Z}$
$\int \frac{1}{1-x^2} dx = \operatorname{argtgh} x + C$	$-1 < x < 1$
$\int \frac{1}{1-x^2} dx = \operatorname{argcotgh} x + C$	$\lvert x \rvert > 1$
$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C_1 = -\arccos x + C_2$	$(-1, 1)$
$\int \frac{1}{\sqrt{1+x^2}} dx = \operatorname{argsinh} x + C = \ln(x + \sqrt{x^2+1}) + C$	$\mathbb{R}$
$\int \frac{1}{1+x^2} dx = \operatorname{arctg} x + C - 1 = -\operatorname{arccotg} x + C_2$	$\mathbb{R}$
$\int \frac{1}{\sqrt{x^2-1}} dx = \operatorname{argcosh} \lvert x \rvert \operatorname{sign} x + C$	$\mathbb{R} - [-1, 1]$
$\int \cosh x dx = \sinh x + C$	$\mathbb{R}$
$\int \sinh x dx = \cosh x + C$	$\mathbb{R}$
$\int \operatorname{tgh} x dx = \ln(\cosh x) + C$	$\mathbb{R}$
$\int \operatorname{cotgh} x dx = \ln \lvert \sinh x \rvert + C$	$\mathbb{R} - \{0\}$

Nekonečné číselné řady

$$\sum \alpha a_n = \alpha \sum a_n \qquad \sum (a_n + b_n) = \sum a_n + \sum b_n$$

Nutná podmínka konvergence: $\lim_{n \to +\infty} a_n = 0$

Bolzano-Cauchyova podmínka konvergence:

$$\sum a_n \quad K \Leftrightarrow \forall \varepsilon > 0 \ \exists \ n_0 \ \forall \ n > n_0, \ p \in \mathbb{N} : \left| \sum_{k=n+1}^{n+p} a_k \right| < \varepsilon$$

Nelimitní tvar srovnávacího kritéria: $0 \leq a_n \leq b_n \ \forall \ n \in \mathbb{N} :$

$$\sum b_n \quad K \Rightarrow \sum a_n \quad K, \qquad \sum a_n \quad D \Rightarrow \sum b_n \quad D$$

Limitní tvar srovnávacího kritéria: $0 \leq a_n, \ 0 \leq b_n \ \forall \ n, \ c = \lim_{n \to +\infty} \frac{a_n}{b_n} :$

$$c \in \mathbb{R} \ \sum b_n \quad K \Rightarrow \sum a_n \quad K, \qquad c \ > \ 0 \ \sum b_n \quad D \Rightarrow \sum a_n \quad D$$

Odmocninové (Cauchyovo) krit.: $\forall \ n \ a_n \geq 0 :$ 1. $\exists q \in \langle 0, 1)$, $\forall \ n \in \mathbb{N} :$ $\sqrt[n]{a_n} \leq q \Rightarrow \sum a_n \ K.$

Pro nekonečně mnoho indexů platí: $\sqrt[n]{a_n} \geq 1 \Rightarrow \sum a_n \ D$

2. $\lim_{n \to +\infty} \sqrt[n]{a_n} < 1 \Rightarrow \sum a_n \ K.$ $\qquad \lim_{n \to +\infty} \sqrt[n]{a_n} > 1 \Rightarrow \sum a_n \ D$

Podílové (d'Alembertovo) krit.: $\forall\ n\ a_n \geq 0:$ 1. $\exists q \in (0,1)\,,\ \forall\ n \in \mathbb{N}: \dfrac{a_{n+1}}{a_n} \leq q \Rightarrow \sum a_n\ K$

$$\forall\ n \in \mathbb{N}: \frac{a_{n+1}}{a_n} \geq 1 \Rightarrow \sum a_n\ D$$

$$2.\ \lim \frac{a_{n+1}}{a_n} < 1 \Rightarrow \sum a_n\ K. \qquad \lim \frac{a_{n+1}}{a_n} > 1 \Rightarrow \sum a_n\ D$$

Integrální kritérium: nechť f - nezáporná, kelsající na $\langle 1, +\infty)$: $\sum f(n)\ K \Leftrightarrow \int_1^{+\infty} f\ K$

Raabeovo kritérium:$\forall\ n\ a_n \geq 0:$ 1. $\exists\ r > 1,\ \forall\ n \in \mathbb{N}: n\left(1 - \dfrac{a_{n+1}}{a_n}\right) \geq r \Rightarrow \sum a_n\ K$

$$\forall\ n \in \mathbb{N}: n\left(1 - \frac{a_{n+1}}{a_n}\right) \leq 1 \Rightarrow \sum a_n\ D$$

$$2. \lim n\left(1 - \frac{a_{n+1}}{a_n}\right) > 1 \Rightarrow \sum a_n\ K \qquad \lim n\left(1 - \frac{a_{n+1}}{a_n}\right) < 1 \Rightarrow \sum a_n\ D$$

Gaussovo kritérium $\forall\ n\ a_n \geq 0:\ \exists\ \lambda, \mu, \alpha \in \mathbb{R}, (c_n)$ omezená : $\forall\ n \in \mathbb{N}: \dfrac{a_{n+1}}{a_n} = \lambda - \dfrac{\mu}{n} - \dfrac{c_n}{n^{1+\alpha}}$

$$1. \lambda < 1 \vee (\lambda = 1 \wedge \mu > 1) \Rightarrow \sum a_n\ K$$

$$2. \lambda > 1 \vee (\lambda = 1 \wedge \mu \leq 1) \Rightarrow \sum a_n\ D$$

Leibnitzovo kritérium: (b_n) klesající posl. kladných č. : $\sum (-1)^{n-1} b_n\ K \Leftrightarrow \lim_{n \to +\infty} b_n = 0$

Modifikované Gaussovo kritérium: $\forall\ n\ b_n > 0:\ \exists\ \lambda, \mu, \alpha \in \mathbb{R}^+, (c_n)$ omezená:

$$\forall\ n \in \mathbb{N}: \frac{b_{n+1}}{b_n} = \lambda - \frac{\mu}{n} - \frac{c_n}{n^{1+\alpha}}:\ 1. \lambda < 1 \vee (\lambda = 1 \wedge \mu > 1) \Rightarrow \sum (-1)^{n-1} b_n\ |K|$$

$$2. \lambda = 1 \wedge (0 < \mu \leq 1) \Rightarrow \sum (-1)^{n-1} b_n\ K$$

$$3. \lambda > 1 \vee (\lambda = 1 \wedge \mu \leq 1) \Rightarrow \sum a_n\ D$$

Dirichlet: $\sum b_n$ omezená posl. část. součtů, (a_n) – monotónní, $\lim a_n = 0 \Rightarrow \sum a_n b_n\ K$

Abel: $\sum b_n\ K, (a_n)$ – omezená, monotónní $\Rightarrow \sum a_n b_n\ K$