BRST-BV Quantum Actions for Constrained Totally-Symmetric Integer HS Fields

Čestmir Burdík[a]*, Alexander A. Reshetnyak[b,c,d]†

[a] *Department of Mathematics, Czech Technical University, Prague 12000, Czech Republic,*

[b] *Tomsk State Pedagogical University, Tomsk 634041, Russia,*

[c] *National Research Tomsk State University, Tomsk 634050, Russia,*

[d] *Institute of Strength Physics and Materials Science SB RAS, Tomsk 634055, Russia*

Abstract

A constrained BRST–BV Lagrangian formulation for totally symmetric massless HS fields in a d-dimensional Minkowski space is extended to a non-minimal constrained BRST–BV Lagrangian formulation by using a non-minimal BRST operator $Q_{c|\mathrm{tot}}$ with non-minimal Hamiltonian BFV oscillators $\overline{C}, \overline{\mathcal{P}}, \lambda, \pi$, as well as antighost and Nakanishi-Lautrup tensor fields, in order to introduce an admissible self-consistent gauge condition. The gauge-fixing procedure involves an operator gauge-fixing BRST-BFV Fermion Ψ_H as a kernel of the gauge-fixing BRST–BV Fermion functional Ψ, manifesting the concept of BFV–BV duality. A Fock-space quantum action with non-minimal BRST-extended off-shell constraints is constructed as a shift of the total generalized field-antifield vector by a variational derivative of the gauge-fixing Fermion Ψ in a total BRST–BV action $S^{\Psi}_{0|s} = \int d\eta_0 \langle \chi^{\Psi 0}_{\mathrm{tot}|c} \big| Q_{c|\mathrm{tot}} \big| \chi^{\Psi 0}_{\mathrm{tot}|c} \rangle$. We use a gauge condition which depends on two gauge parameters, thereby extending the case of R_ξ-gauges. For the generating functionals of Green's functions, BRST symmetry transformations are suggested and Ward identity is obtained.

1 Introduction

Many of the topical issues in high-energy physics are related to higher-spin (HS) field theory as part of the LHC experiment program. Various tensionless limits of (super)string theory [1] implied by their respective BRST operators contain an infinite set of HS fields with integer and half-integer generalized spins, as well as a set of HS fields with continuous generalized spin [2], [3] (for another viewpoint, see [4]). This incorporates HS field theory into superstring theory and transforms it into a method for studying the classical and quantum structures of the latter (for the present status of HS field theory, see the reviews

*burdik@kmlinux.fjfi.cvut.cz †reshet@tspu.edu.ru

[5], [6], [7], [8]). One of the most efficient ways of studying HS field dynamics, including Lagrangian formulations (LF) in constant curvature spaces and starting from the initial unitary irreducible representation (UIR) of the Poincaré or anti-de-Sitter groups, is based on a constrained BRST–BFV approach for lower integer spins, examined for the first time in [9], [10] and originated from the so-called minimal part of the BFV method [11], [12], devised for quantizing constrained dynamical systems. The BRST–BFV approach is intended to solve the inverse problem, which is formulating an LF in terms of Hamiltonian-like objects by using an auxiliary Hilbert space whose vectors consist of HS (spin)-tensor fields. Incorporating holonomic (traceless and mixed-symmetry) constraints, together with differential ones, into a total system of constraints (without any restrictions imposed on the entire set of the initial and auxiliary HS fields) – which is to be closed with respect to Hermitian conjugation supplied with an appropriate conversion procedure for the subsystem of second-class constraints – has resulted in augmenting the original method by an unconstrained BRST–BFV method. Applications of this method have been originated by [13] and followed by [14], [15], [16], [17], [18] for totally-symmetric HS fields and mixed-(anti)symmetric HS fields in $R^{1,d-1}$ and AdS_d [19], [20], [21], [22], [23], [24]; for a review and the interaction problem, see [7]. A detailed correspondence between the constrained and unconstrained BRST–BFV methods for arbitrary massless and massive IR of the $ISO(1, d-1)$ group with a generalized discrete spin has been recently studied in [25], where equivalence between the underlying constrained and unconstrained LF was established. A development of this topic has resulted in an (un)constrained BRST–BV method of finding minimal BV actions required to construct a quantum action in the BV quantization [29] presented by [30]; for bosonic HS fields, see also [26], [27], [28]. Recently, the issues of LF construction and dynamics for continuous spin particles (CSP) have been analyzed in the Shuster–Toro representation for bosonic [31] and fermionic [32] fields by R. Metsaev [33] and also in [34] using the Weyl spinor notation – for recent developments, see also [35], [36], [37], [38], [39], [40] – whereas constrained BRST–BFV and BRST–BV descriptions of CSP particle dynamics using the original Wigner–Bargmann equations [41] have been examined in [42], along with a special tensionless limit of string theory.

The quantization problem lies in constructing a so-called Batalin–Vilkovisky action for (general) reducible gauge-invariant LF. Until now, the problem has been solved by representing BRST-BFV or BRST–BV Lagrangians in component or oscillator forms without Hamiltonian operator oscillators in the minimal sector [11], [12], following the BV method [29] (or its simplified version of the Faddeev–Popov recipe [43] for irreducible theories with closed algebras), as was done, for instance, in [44]; see also [45]. At the same time (because the set of all monomials in the powers of minimal Hamiltonian oscillators is in one-to-one correspondence with the set of all field and antifield vectors) the set of non-minimal Hamiltonian oscillators of the BFV method, as well as the set of antighost $|\overline{C}\rangle$ and Nakanishi–Lautrup $|b\rangle$ field vectors from the non-minimal sector of the BV field-antifield space, has not been utilized to form the field $|\overline{C}\rangle$, $|b\rangle$ and the related antifield vectors. This fact has prevented one from imposing admissible gauge-fixing condition as a respective shift of antifields in the BV action. Having in mind an equivalence between unconstrained and constrained BRST–BFV LF for one and the same HS field of generalized discrete spin in $\mathbb{R}^{1,d-1}$ [25], we restrict ourselves by a massless totally-symmetric (TS) HS field with integer helicities $s \in \mathbb{N}_0$ in a constrained BRST-BFV LF. The paper is devoted to the following problems:

1. Enlargement of constrained BRST–BV Lagrangians for TS massless fields with integer

helicity in a d-dimensional Minkowski space-time to non-minimal constrained BRST–BV Lagrangians with a compatible set of off-shell BRST-extended constraints in the metric formulation;

2. Introduction of a gauge-fixing procedure in order to eliminate the gauge degeneracy of a classical BRST–BFV action so as to construct a quantum (non-renormalized) BRST–BV action and a generating functional of Green's functions with an essential self-consistent use of variables from non-minimal sectors originated from both the operator BFV and functional BV methods.

The paper is organized as follows. In Section 2, we overview the constrained BRST–BV and BRST–BFV approaches in the minimal sector of field-antifield variables for a TS HS field of integer helicity. The construction of a total BV action on the basis of a constrained BRST–BV approach augmented by respective non-minimal field-antifield tensor fields and non-minimal Hamiltonian oscillators for the TS HS fields is considered in Section 3. In Section 4, we present a gauge-fixing procedure used to construct quantum BV actions with an underlying BRST symmetry for respective generating functionals of Green's functions, and make concluding comments in Section 5.

We use the convention $\eta_{\mu\nu} = diag(+, -, ..., -)$ for the metric tensor, with Lorentz indices $\mu, \nu = 0, 1, ..., d-1$, and the respective notation $\epsilon(F)$, $[gh_H, gh_L, gh_{tot}](F)$ for the value of Grassmann parity and those of the BFV, gh_H, BV, gh_L and total $gh_{tot} = gh_H + gh_L$ ghost number of a homogeneous quantity F. The supercommutator $[F, G\}$ of any quantities F and G with definite values of Grassmann parity is given by $[F\,, G\} = FG - (-1)^{\epsilon(F)\epsilon(G)}GF$.

2 Constrained BRST-BFV and BRST-BV Lagrangians

Recall that the UIR of $ISO(1, d-1)$ group with of zero mass and integer helicity $s \in \mathbb{N}_0$ is realized using an $\mathbb{R}$-valued TS tensor field $\phi_{\mu_1 \ldots \mu_s}(x) \equiv \phi_{(\mu)_s}$ and described by the following equivalent conditions

$$\big(\partial^\nu\partial_\nu,\, \partial^{\mu_1},\, \eta^{\mu_1\mu_2}\big)\phi_{(\mu)_s} = (0, 0, 0) \iff \big(l_0,\, l_1,\, l_{11}, g_0 - d/2\big)|\phi\rangle = (0, 0, 0, s)|\phi\rangle \tag{1}$$

for a basic vector and operators in a Fock space $\mathcal{H}$ generated by Grassmann-even (symmetric basis) oscillators a_μ, a^+_ν ($[a_\mu, a^+_\nu] = -\eta_{\mu\nu}$)

$$|\phi\rangle = \sum_{s\geq 0} \frac{\imath^s}{s!}\phi^{(\mu)_s} \prod_{i=1}^{s} a^+_{\mu_i}|0\rangle, \quad \big(l_0,\, l_1,\, l_{11}, g_0\big) = \big(\partial^\nu\partial_\nu,\, -\imath a^\nu\partial_\nu,\, \frac{1}{2}a^\mu a_\mu, -\frac{1}{2}\{a^+_\mu . a^\mu\}\big). \tag{2}$$

The constrained BRST-BFV approach for a free TS massless HS field $\phi^{(\mu)_s}$ in Minkowski space results in an irreducible gauge-invariant LF with a nilpotent hermitian constrained BRST operator Q_c, an off-shell BRST-extended traceless constraint $\widehat{L}_{11}$ and a spin operator $\widehat{\sigma}_c(g)$ acting in a total Hilbert space $\mathcal{H}_{c|tot} = \mathcal{H}\otimes H^{o_a}_{gh}$ with scalar product $\langle\bullet|\bullet\rangle$, which admits a $\mathbb{Z}$-grading $\mathcal{H}_{c|tot} = \oplus_e \mathcal{H}^e_{c|tot}$ corresponding to the ghost number gh_H, $gh_H(\mathcal{H}^e_{c|tot}) = -e$:

$$\mathcal{S}_{C|s}(\phi, \phi_1, \phi_2) = \int d\eta_{0s}\langle\chi^0_c|Q_c|\chi^0_c\rangle_s,\;\; \delta|\chi^0_c\rangle_s = Q_c|\chi^1_c\rangle_s, \quad (\epsilon, gh_H)|\chi^e_c\rangle = (e, -e), \tag{3}$$

$$Q_c = \eta_0 l_0 + \eta^+_1 l_1 + l^+_1\eta_1 + \imath\eta^+_1\eta_1\mathcal{P}_0 \;=\; \eta_0 l_0 + \Delta Q_c + \imath\eta^+_1\eta_1\mathcal{P}_0, \quad (\epsilon, gh_H)Q_c = (1, 1), \tag{4}$$

$$\big(\widehat{L}_{11},\, \widehat{\sigma}_c\big)|\chi^e_c\rangle_s = \Big(0,\, s + \frac{d-2}{2}\Big)|\chi^e_c\rangle_s, \quad \big(\widehat{L}_{11},\, \widehat{\sigma}_c\big) = \big(l_{11} + \eta_1\mathcal{P}_1,\, g_0 + \eta^+_1\mathcal{P}_1 - \eta_1\mathcal{P}^+_1\big) \tag{5}$$

for $e=0,1$ and $|\chi^e_c\rangle_s \in \mathcal{H}^e_{c|tot}$. Here, the subspace $H^{o_a}_{gh}$ is generated by some additional (to a_μ, a^+_μ) BFV Grassmann-odd ghost operators from the minimal sector $\{C^a, \mathcal{P}_a\} = \{\eta_0, \mathcal{P}_0; \eta_1, \mathcal{P}^+_1; \eta^+_1, \mathcal{P}_1\}$, with the ghost number distribution $gh_H(\eta) = -gh_H(\mathcal{P}) = 1$ and the non-vanishing anticommutators $\{\eta_0, \mathcal{P}_0\} = \imath$, $\{\eta_1, \mathcal{P}^+_1\} = 1$. These ghost operators are introduced for the system of first-class differential constraints $\{o_a\} = \{l_0, l_1, l^+_1\}$: $l^+_1 = -\imath a^{+\nu}\partial_\nu$ subject to the algebra: $[l_0, l^{(+)}_1] = 0$, $[l_1, l^+_1] = l_0$. The operators Q_c, $\widehat{L}_{11}$, $\widehat{\sigma}_c$ are found as solutions of the generating equations [25]

$$Q^2_c = 0, \quad [Q_c, \widehat{L}_{11}\} = 0, \quad [Q_c, \widehat{\sigma}_c\} = 0, \quad [\widehat{L}_{11}, \widehat{\sigma}_c\} = 2\widehat{L}_{11}. \tag{6}$$

The field $|\chi^0_c\rangle_s$ and the gauge parameter $|\chi^1_c\rangle_s$ labelled by the symbol "s" as eigenvectors of the spin condition in (5) read as follows ($\phi_2(a^+)\rangle \equiv 0$ when $s \le 1$ and $\phi_1(a^+)\rangle \equiv 0$ for $s=0$):

$$|\chi^0_c\rangle_s = |S_c\rangle_s + \eta_0|B_c\rangle_s = |\phi(a^+)\rangle_s + \eta^+_1\mathcal{P}^+_1|\phi_2(a^+)\rangle_{s-2} + \eta_0\mathcal{P}^+_1|\phi_1(a^+)\rangle_{s-1} \tag{7}$$

$$= \Big(\frac{\imath^s}{s!}\phi^{(\mu)_s}\prod_{i=1}^{s}a^+_{\mu_i} + \eta^+_1\mathcal{P}^+_1\frac{\imath^{s-2}}{(s-2)!}\phi_2^{(\mu)_{s-2}}\prod_{i=1}^{s-2}a^+_{\mu_i} + \eta_0\mathcal{P}^+_1\frac{\imath^{s-1}}{(s-1)!}\phi_1^{(\mu)_{s-1}}\prod_{i=1}^{s-1}a^+_{\mu_i}\Big)|0\rangle, \tag{8}$$

$$|\chi^1_c\rangle_s = \mathcal{P}^+_1|\xi(a^+)\rangle_{s-1} = \mathcal{P}^+_1\frac{\imath^{s-1}}{(s-1)!}\xi_{(\mu)_{s-1}}\prod_{i=1}^{s-1}a^{+\mu_i}|0\rangle. \tag{9}$$

Solving the traceless constraints (5) leads to the following relations for the fields:

$$\begin{aligned}&l_{11}\left(|\phi\rangle; |\phi_1\rangle, |\phi_2\rangle, |\xi\rangle\right) = \left(-|\phi_2\rangle; 0,0,0\right) \iff \\ &\Big(\phi^{(\mu)_{s-2}\mu}{}_\mu; \phi_1^{(\mu)_{s-3}\mu}{}_\mu, \phi_2^{(\mu)_{s-4}\mu}{}_\mu, \xi^{(\mu)_{s-3}\mu}{}_\mu\Big) = \Big(2\phi_2^{(\mu)_{s-2}}; 0,0,0\Big).\end{aligned} \tag{10}$$

The gauge-invariant action $\mathcal{S}_{C|s} = \mathcal{S}_{C|s}(\phi, \phi_1, \phi_2)$ is written in the triplet: η_0-independent, ghost-independent and tensor forms [49]

$$\mathcal{S}_{C|s} = \big({}_s\langle S_c\big|{}_s\langle B_c\big|\big)\begin{pmatrix} l_0 & -\Delta Q_c \\ -\Delta Q_c & \eta^+_1\eta_1\end{pmatrix}\begin{pmatrix}\big|S_c\rangle_s \\ \big|B_c\rangle_s\end{pmatrix}, \delta\begin{pmatrix}\big|S_c\rangle_s \\ \big|B_c\rangle_s\end{pmatrix} = \begin{pmatrix}\Delta Q_c \\ l_0\end{pmatrix}\big|S^1\rangle_s, \tag{11}$$

$$\mathcal{S}_{C|s} = \big({}_s\langle\phi\big|{}_{s-2}\langle\phi_2\big|{}_{s-1}\langle\phi_1\big|\big)\begin{pmatrix} l_0 & 0 & -l^+_1 \\ 0 & -l_0 & l_1 \\ -l_1 & l^+_1 & 1\end{pmatrix}\begin{pmatrix}\big|\phi\rangle_s \\ \big|\phi_2\rangle_{s-2} \\ \big|\phi_1\rangle_{s-1}\end{pmatrix}, \tag{12}$$

$$\delta\left(\big|\phi\rangle_s, \big|\phi_1\rangle_{s-1}, \big|\phi_2\rangle_{s-2}\right) = \left(l^+_1, l_0, l_1\right)|\xi\rangle_{s-1}\,, \tag{13}$$

$$\begin{aligned}\mathcal{S}_{C|s} = \frac{(-1)^s}{s!}\int d^dx\Big\{&\phi_{(\mu)_s}\big(\partial^2\phi^{(\mu)_s} + 2s\partial^{\mu_s}\phi_1^{(\mu)_{s-1}}\big) - s(s-1)\phi_{2(\mu)_{s-2}}\partial^2\phi_2^{(\mu)_{s-2}} \\ &- s\phi_{1(\mu)_{s-1}}\big(\phi_1^{(\mu)_{s-1}} - 2(s-1)\partial^{\mu_{s-1}}\phi_2^{(\mu)_{s-2}}\big)\Big\},\end{aligned} \tag{14}$$

$$\delta\big(\phi^{(\mu)_s}, \phi_1^{(\mu)_{s-1}}, \phi_2^{(\mu)_{s-2}}\big) \;=\; \big(-\partial^{(\mu_s}\xi^{(\mu)_{s-1})}, \partial^2\xi^{(\mu)_{s-1}}, \partial_{\mu_s}\xi^{(\mu)_{s-1}}\big)$$

(for $|S^1\rangle_s \equiv |\chi^1_c\rangle_s$) as well as in the doublet form with $\mathcal{S}^d_{C|s} = \mathcal{S}_{C|s}\big|_{\left(\phi_1=\phi_1(\phi,\phi_2)\right)}$ (having expressed $\big|\phi_1\rangle_{s-1}$ from the equation of motion: $\big|\phi_1\rangle = l_1\big|\phi\rangle - l^+_1\big|\phi_2\rangle$)

$$\mathcal{S}^d_{C|s} = \big({}_s\langle\phi\big|{}_{s-2}\langle\phi_2\big|\big)\begin{pmatrix} l_0 - l^+_1 l_1 & (l^+_1)^2 \\ l^2_1 & -l_0 - l_1 l^+_1\end{pmatrix}\begin{pmatrix}\big|\phi\rangle_s \\ \big|\phi_2\rangle_{s-2}\end{pmatrix}, \tag{15}$$

and in the single field form (Fronsdal [48]):

$$\mathcal{S}_{F|s}(\phi) = {}_s\langle\phi\big|\left(l_0 - l_1^+ l_1 - (l_1^+)^2 l_{11} - l_{11}^+ l_1^2 - l_{11}^+(l_0 + l_1 l_1^+) l_{11}\right)\big|\phi\rangle_s, \qquad (16)$$

$$\delta\big|\phi\rangle_s = l_1^+|\xi\rangle_{s-1} \quad \text{and} \quad l_{11}\big(l_{11}|\phi\rangle,\, |\xi\rangle\big) = (0,0), \qquad (17)$$

$$\mathcal{S}_{F|s}(\phi) = \frac{(-1)^s}{s!}\int d^dx\Big\{\phi_{(\mu)_s}\big(\partial^2\phi^{(\mu)_s} - s\partial^{\mu_s}\partial_\nu\phi^{(\mu)_{s-1}\nu} + s(s-1)\partial^{\mu_{s-1}}\partial^{\mu_s}\phi^{(\mu)_{s-2}\mu}{}_\mu\big)$$
$$-\frac{1}{2}s(s-1)\phi_{(\mu)_{s-2}\mu}{}^\mu\big(\partial^2\phi^{(\mu)_{s-2}\nu}{}_\nu + \frac{1}{2}(s-2)\partial^{\mu_{s-2}}\partial^\mu\phi^{(\mu)_{s-3}}{}_{\mu\nu}{}^\nu\big)\Big\}. \qquad (18)$$

In its turn, the constrained BRST–BV method of finding a BV action in the minimal sector of field-antifield variables augments the BRST–BFV algorithm by transforming the gauge parameter $|\chi_c^1\rangle$ into a ghost field $|C\rangle$, thereby introducing the respective antifields $|\chi_c^{*0}\rangle$, $|C^*\rangle$ and incorporating a unique generalized field-antifield vector.

Depending on a given BRST–BFV LF (triplet, doublet, or Fronsdal), the extension of the configuration space $\mathcal{M}_{\rm cl}$, due to other HS tensors, up to a minimal configuration space $\mathcal{M}_{\rm min}$ in the case of a TS field is determined by a *generalized vector* $|\chi_{\mathrm{g}|c}\rangle$ from a generalized Hilbert space $\mathcal{H}_{g|tot} = \mathcal{H}_g \otimes H_{gh}^{o_a}$ (instead of $|\chi_c\rangle \in \mathcal{H}_{c|tot}$),

$$|\chi_{\mathrm{g}|c}\rangle \;=\; \sum_n \frac{\imath^n}{n!}\Big(\prod(\eta_0)^{n_{f0}}\prod(\eta_1^+)^{n_f}(\mathcal{P}_1^+)^{n_p}\phi^{n_{f0}n_f n_p}_{\mathrm{g}|c(\mu)_n}\prod_{i=1}^n a^{\mu_i+}\Big)|0\rangle, \qquad (19)$$

for $|\phi(a^+)^{n_{f0}n_f n_p}_{\mathrm{g}|c}\rangle \equiv \phi^{n_{f0}n_f n_p}_{\mathrm{g}|c(\mu)_n}\prod_{i=1}^n a^{\mu_i+}|0\rangle$. Here any ghost independent vector $\phi(a^+)^{\cdots}_{\mathrm{g}|c}|0\rangle \in \mathcal{H}_g$ with a vanishing BV ghost number $[gh_L(\phi(a^+)^{\cdots}_{\mathrm{g}|c}) = 0]$ coincides with the field vector $\phi(a^+)^{\cdots}_c|0\rangle \in \mathcal{H}_c$ for the vector $|\chi_c\rangle$ also determined by the representation (19). The space $\mathcal{H}_{g|tot}$ admits a $\mathbb{Z}\oplus\mathbb{Z}$-grading corresponding to the ghost numbers gh_H, gh_L distributions

$$\mathcal{H}_{g|tot} = \oplus_{e,l}\mathcal{H}^{e,l}_{g|tot}: \quad gh_H(\mathcal{H}^{e,l}_{g|tot}) = -e,\; gh_L(\mathcal{H}^{e,l}_{g|tot}) = l \qquad (20)$$

From the same spectral problem for Q_c-complex with imposing of spin and BRST-extended constraint in $\mathcal{H}_{g|tot}$:

$$Q_c|\chi^0_{\mathrm{g}|c}\rangle = 0, \quad \delta|\chi^l_{\mathrm{g}|c}\rangle = Q_c|\chi^{l+1}_{\mathrm{g}|c}\rangle, \quad (\widehat{L}_{11},\, \widehat{\sigma}_c)|\chi^l_{\mathrm{g}|c}\rangle = \Big(0,\, s + \frac{d-2}{2}\Big)|\chi^l_{\mathrm{g}|c}\rangle \qquad (21)$$

albeit with a vanishing total ghost number for $|\chi^0_{\mathrm{g}|c}\rangle$, (i.e. without using the sequence of gauge transformations: $\delta|\chi^l_{\mathrm{g}|c}\rangle = Q_c|\chi^{l+1}_{\mathrm{g}|c}\rangle = 0$, for $l = 0,1,...$) we obtain the spin and modified ghost numbers distributions for proper eigen-vector $|\chi^0_{\mathrm{g}|c}\rangle_s$: $gh_{tot}(|\chi^0_{\mathrm{g}|c}\rangle_s) = 0$. This means, that $|\chi^0_{\mathrm{g}|c}\rangle \in \mathcal{H}^0_{g|tot}$ for $\mathcal{H}^p_{g|tot} \equiv \oplus_{e+l=p}\mathcal{H}^{e,l}_{g|tot}$. The whole set of fields $\phi^A_{\rm min}$ and antifields $\phi^*_{A\,\rm min}$ $[(\epsilon, gh_L)\phi^*_{A\,\rm min} = (\epsilon(\phi^A_{\rm min}) + 1, -gh_L(\phi^A_{\rm min}) - 1)]$ from the minimal BV sector for given BRST-BFV (triplet) LF with the use of the condensed notations:

$$\{\phi^A_{\rm min}\} = \{\phi_k^{(\mu)_{s-k}}, C^{(\mu)_{s-1}}\}(x), \quad \{\phi^*_{A\,\rm min}\} = \{\phi^*_{k(\mu)_{s-k}}, C^*_{(\mu)_{s-1}}\}(x), \qquad (22)$$

is in one-to-one correspondence with the set of field components in the *minimal generalized vector* of spin s

$$|\chi^0_{\mathrm{g}|c}\rangle_s \;=\; |\chi^0_c\rangle_s + \mathcal{P}_1^+ C_{s-1}(a^+)|0\rangle + |\chi^{0*}_c\rangle_s - \eta_0\eta_1^+ C^*_{s-1}(a^+)|0\rangle. \qquad (23)$$

$$|\chi^{0*}_c\rangle_s \;=\; |S^*_c\rangle_s + \eta_0|B^*_c\rangle_s \;=\; \eta_1^+|\phi_1^*(a^+)\rangle_{s-1} + \eta_0|\phi^*(a^+)\rangle_s + \eta_0\mathcal{P}_1^+\eta_1^+|\phi_2^*(a^+)\rangle_{s-2}. \qquad (24)$$

	$a^{(+)}$	C^a	$\mathcal{P}_a$	$\phi_k^{(\mu)_{s-k}}$	$C^{(\mu)_{s-1}}$	$\phi^*_{k(\mu)_{s-k}}$	$C^*_{(\mu)_{s-1}}$
gh_H	0	1	-1	0	0	0	0
gh_L	0	0	0	0	1	-1	-2
$gh_{\rm tot}$	0	1	-1	0	1	-1	-2
ϵ	0	1	1	0	1	1	0

Table 1: Ghost numbers and Grassman parity distributions.

The properties of $\mathbb{Z}$ gradings for all BRST-BV oscillators and (anti)field variables are presented in Table 1. The *classical antifield* $|\chi^{0*}_c\rangle_s$ and ghost field vectors $|C\rangle_s$ are naturally constructed from the *classical field* vector $|\chi^0_c\rangle_s$ and the gauge parameter $|\chi^1_c\rangle_s$, with the spin and ghost number relations preserved on a basis of the correspondence

$$\eta_0^{n_{f0}}\eta_1^{+n_f}\mathcal{P}_1^{+n_p}|\phi(a^+)_c^{n_{f0}n_fn_p}\rangle \to \eta_0^{(n_{f0}+1)\mathrm{mod}2}\mathcal{P}_1^{+n_f}\eta_1^{+n_p}|\phi^*(a^+)_c^{n_{f0}n_fn_p}\rangle, \tag{25}$$
$$|\chi^1_c\rangle_s \; = \; |C\rangle_s\mu \;\texttt{ for }\; (gh_H, gh_L, gh_{tot})\mu \; = \; (0,-1,-1). \tag{26}$$

The minimal BV actions with account of the constrained and complex conjugation properties for the ghost field $C^{(\mu)_{s-1}}$ and antifields,

$$l_{11}\left(|\phi^*\rangle;|\phi^*_1\rangle,|\phi^*_2\rangle,|C^{(*)}\rangle\right) = (|\phi^*_2\rangle;0,0,0), \tag{27}$$
$$\left(C^{(\mu)_{s-1}}\right)^+ = C^{(\mu)_{s-1}}, \quad \left(\phi^*_{k(\mu)_{s-k}}\right)^+ = (-1)^k\phi^*_{k(\mu)_{s-k}}, \; k=0,1,2 \tag{28}$$

in the general form, as well as the η_0-independent, component, tensor-triplet $S_{C|s}$, doublet $S^d_{C|s}$, and Fronsdal-like $S_{F|s}$ forms read as follows:

$$S_{C|s} = \int d\eta_0 \; {}_s\langle\chi^0_{\mathrm{g}|c}|Q_c|\chi^0_{\mathrm{g}|c}\rangle_s \; = \; \mathcal{S}_{C|s} + \int d\eta_0 \left\{ {}_s\langle\chi^{0*}_c|Q_c|C\rangle_s + {}_s\langle C|Q_c|\chi^{0*}_c\rangle_s\right\} \tag{29}$$
$$= \mathcal{S}_{C|s} + \left({}_s\left(\langle S^*_c|\langle B^*_c|\right) \begin{pmatrix} l_0 & -\Delta Q_c \\ -\Delta Q_c & \eta_1^+\eta_1 \end{pmatrix} \begin{pmatrix} |C\rangle_s \\ 0 \end{pmatrix} + h.c.\right) \tag{30}$$
$$= \mathcal{S}_{C|s} + \left(\left[{}_{s-1}\langle\phi^*_1|l_0 - {}_s\langle\phi^*_0|l_1^+ - {}_{s-2}\langle\phi^*_2|l_1\right] C_{s-1}(a^+)|0\rangle + h.c.\right) \tag{31}$$
$$= \mathcal{S}_{C|s} + 2\frac{(-1)^s}{s!}\int d^dxs\left(\phi^*_{(\mu)_s}\partial^{\mu_s} + \phi^*_{1(\mu)_{s-1}}\partial^2 - \phi^*_{(\mu)_{s-2}}\partial_{\mu_{s-1}}\right)C^{(\mu)_{s-1}}; \tag{32}$$
$$S^d_{C|s} = \mathcal{S}^d_{C|s} - \left(\left[{}_s\langle\phi^*_0|l_1^+ + {}_{s-2}\langle\phi^*_2|l_1\right] C_{s-1}(a^+)|0\rangle + h.c.\right); \tag{33}$$
$$S_{F|s}(\phi,\phi^*,C) = \mathcal{S}_{F|s}(\phi) + 2\frac{(-1)^s}{s!}\int d^dx\phi^*_{(\mu)_s}\partial^{\{\mu_s}C^{(\mu\})_{s-1}}. \tag{34}$$

The functional $S_{C|s}$ is invariant with respect to the minimal Lagrangian BRST-like transformations (with a Grassmann-odd constant parameter μ) for the field vectors $|\chi^0_c(x)\rangle, |C(x)\rangle$

$$\delta_\mu|\chi^0_c(x)\rangle_s \;=\; \mu\frac{\overrightarrow{\delta}}{\delta_s\langle\chi^*_c(x)|}S_{C|s} \;=\; \mu Q_c|C(x)\rangle_s, \tag{35}$$
$$\delta_\mu|C(x)\rangle_s \;=\; \mu\frac{\overrightarrow{\delta}}{\delta_s\langle C^*(x)|}S_{C|s} \;=\; 0, \quad \epsilon\left(\frac{\overrightarrow{\delta}}{\delta_s\langle\chi^*_c(x)|}, \frac{\overrightarrow{\delta}}{\delta_s\langle C^*(x)|}\right) = 1, \tag{36}$$

with constant antifields $\delta_B|\chi_c^{0*}\rangle = 0$ (as well as for the duals $\langle\chi_c^0(x)|, \langle C(x)|$) or, equivalently, in terms of a *new BRST-like generator* $\overrightarrow{s}_0$ and its dual $\overleftarrow{s}_0$:

$$\delta_B\left[|\chi_c^0(x)\rangle_s, |C(x)\rangle_s\right] = \mu\overrightarrow{s}_0\left[|\chi_c^0(x)\rangle_s, |C(x)\rangle_s\right] = \mu\,[Q_c, 0]\,|C(x)\rangle_s, \quad (37)$$

$$\delta_B\left[{}_s\langle\chi_c^0(x)|, {}_s\langle C(x)|\right] = \left[{}_s\langle\chi_c^0(x)|, {}_s\langle C(x)|\right]\overleftarrow{s}_0\mu = {}_s\langle C(x)|\,[Q_c, 0]\,\mu. \quad (38)$$

Indeed,

$$\delta_B S_{C|s} = \left(\delta_{B\,s}\langle\chi_c^0|\frac{\overrightarrow{\delta}S_{C|s}}{\delta_s\langle\chi_c^0|} + \frac{S_{C|s}\overleftarrow{\delta}}{\delta|\chi_c^0\rangle_s|}\delta_B|\chi_c^0\rangle_s\right) = \mu\Big({}_s\langle C|Q_c^2|\chi_c^0\rangle_s - {}_s\langle\chi_c^0|Q_c^2|C\rangle_s\Big) = 0. \quad (39)$$

The variational derivatives with respect to the vectors $|\chi_c^{0(*)}\rangle_s$, $|C^{(*)}(x)\rangle_s$ and their duals in (35), (36), (39), e.g. for any quadratic (in the fields) functional with the kernel E_F

$$F = \int d\eta_0\mathcal{F}(\chi_c^{0(*)}, C^{(*)}) = \int d\eta_{0s}\langle\chi_{\mathrm{g}|c}^0|E_F|\chi_{\mathrm{g}|c}^0\rangle_s \equiv \int d\eta_0\mathcal{F}(\eta_0) \quad (40)$$

are given in terms of variational derivatives for a fixed η_0 with a vanishing Grassman parity of the density $\mathcal{F}$ ($\epsilon(\mathcal{F}) = \epsilon(E_F) = \epsilon(F) + 1$) according to (23), (24)

$$\left(\frac{F\overleftarrow{\delta}}{\delta|\chi_c^{0(*)}\rangle_s}; \frac{\overrightarrow{\delta}F}{\delta_s\langle\chi_c^{(*)}|}; \frac{F\overleftarrow{\delta}}{\delta|C^*\rangle_s}; \frac{\overrightarrow{\delta}F}{\delta_s\langle C|}\right) = \left(\frac{\mathcal{F}\overleftarrow{\delta}_{\eta_0}}{\delta|\chi_c^{0(*)}\rangle_s}; \frac{\overrightarrow{\delta}_{\eta_0}\mathcal{F}}{\delta_s\langle\chi_c^{0(*)}|}; \frac{\mathcal{F}\overleftarrow{\delta}_{\eta_0}}{\delta|C^*\rangle_s}; \frac{\overrightarrow{\delta}_{\eta_0}\mathcal{F}}{\delta_s\langle C|}\right). \quad (41)$$

For these variational derivatives, the following normalization holds true (for $\delta(\eta_0' - \eta_0) = \eta_0' - \eta_0$):

$$\left(\frac{|A(\eta_0; x)\rangle_s\overleftarrow{\delta}}{\delta|A(\eta_0'; x')\rangle_s}; \frac{\overrightarrow{\delta}_s\langle A(\eta_0; x)|}{\delta_s\langle A(\eta_0'; x')|}\right) = \delta(\eta_0' - \eta_0)\big(\delta(x' - x); \delta(x' - x)\big),\ A \in \{\chi_c^{0(*)}, C_c^{(*)}\}. \quad (42)$$

The BRST-BV actions allow for consistency when deriving interaction vertexes (for developments in the metric-like form see, e.g., [7], [28], [46], [47]). In general (e.g. for the cubic vertex), one considers three independent Hilbert spaces, $\mathcal{H}_{c|tot}^i = \mathcal{H}_i \otimes H_{i|gh}^{o_a}$, $i = 1, 2, 3$, and finds a BRST invariant vertex V in the tensor product $\otimes_{i=1}^3\mathcal{H}_{c|tot}^i$ without any off-shell constraints [46], being the case of a reducible $ISO(1, d-1)$ representation. Restricting ourselves for simplicity of illustration by a cubic self-interacting vertex, $\big|V\rangle_{(s,s,s)} = \big|V(\{\widetilde{l}_0\}, \{\widetilde{l}_1^+\}, \{\widetilde{l}_{11}^+\})\rangle_{(s,s,s)} \equiv \big|V\rangle_{3s}$, we can solve the problem, for instance, by deforming $\mathcal{S}_{F|s}(\phi)$ (16) and the gauge transformations (17) within a gauge model having the same double-traceless field $\phi_{(\mu)_s}$ by means of the self-interaction terms $\mathcal{S}_{1|s}(\phi)$ and by $S_{1g|s} = S_{1g|s}(\phi, \phi^*, C)$, respectively,

$$S_{[1]|s}(\phi, \phi^*, C, C^*) = S_{F|s}(\phi, \phi^*, C) + \mathcal{S}_{1|s}(\phi) + S_{1g|s} + S_{2g|s}(\phi, \phi^*, C, C^*), \quad (43)$$

$$\mathcal{S}_{1|s}(\phi) = g\Big({}_s\langle\phi| \otimes {}_s\langle\phi| \otimes {}_s\langle\phi|\big|V\rangle_{3s} + {}_{3s}\langle V^+|\big|\phi\rangle_s \otimes |\phi\rangle_s \otimes |\phi\rangle_s\Big), \quad (44)$$

$$S_{1g|s} = g\Big({}_s\langle\phi^*| \otimes {}_s\langle\phi|_{s-1}\langle C|\big|V_1\big(\{\widetilde{l}_0\}\{\widetilde{l}_1\}, \{\widetilde{l}_{11}\}\big)\rangle_{3s-1} + {}_{3s-1}\langle V_1^+|\big|C\rangle_{s-1} \otimes |\phi\rangle_s \otimes |\phi^*\rangle_s\Big), (45)$$

$$S_{2g|s} = \frac{g}{2}\Big({}_{s-1}\langle C^*| \otimes {}_{s-1}\langle C| \otimes {}_{s-1}\langle C|\big|F\rangle_{3(s-1)} + {}_{3(s-1)}\langle F^+|\big|C\rangle_{s-1} \otimes |C\rangle_{s-1} \otimes |C^*\rangle_{s-1}\Big). (46)$$

Here, first, the local product $\otimes_{k=1}^{p}|\phi\rangle_s$ (and also the sets $\{\widetilde{l}_0\}$, $\{\widetilde{l}_1\}$, $\{\widetilde{l}_{11}\}$) is understood as

$$\otimes_{k=1}^{p}\big(\phi_{(\mu^k)_s}(x)\prod a_k^{+(\mu^k)_s}|0\rangle\big) \quad \text{and} \quad \{\widetilde{l}_0\} = \{\eta^{\mu\nu}\partial^k_\nu\partial^k_\mu\} \equiv \{l_0^k\},$$
$$\{\widetilde{l}_1\} = \{-\imath a^{\mu,k}\partial^l_\mu\} \equiv \{l_1^{kl}\}, \qquad \{\widetilde{l}_{11}\} = \{\tfrac{1}{2}a^{\mu,k}a^l_\mu\} \equiv \{l_{11}^{kl}\},\ k,l = 1,...,3,$$

with 3 sets of oscillators $a_k^{+\mu}, a_k^\nu$, $[a_k^{+\mu}, a_l^\nu] = \delta_{kl}\eta^{\mu\nu}$. Second, the last summand (46) with an operator quantity $\big|F\rangle_{3(s-1)} \equiv \big|F\big(\{\widetilde{l}_0\}, \{\widetilde{l}_1\}, \{\widetilde{l}_{11}\}\big)\rangle_{3(s-1)}$ is necessary to have the closed deformed algebra of non-Abelian gauge transformations determined with help of field independent term, $\big|V_1\big(\{\widetilde{l}_0\}, \{\widetilde{l}_1\}, \{\widetilde{l}_{11}\}\big)\rangle_{3s-1}$:

$$\delta_{[1]}|\phi\rangle_s = (\delta_0+\delta_1)|\phi\rangle_s = l_1^+|\xi\rangle_{s-1} + g\Big({}_s\langle\phi|\otimes{}_{s-1}\langle\xi| + {}_{s-1}\langle\xi|\otimes{}_s\langle\phi|\Big)\big|V_1\rangle_{3s-1}, \quad (47)$$

$$\big|V_1\rangle_{3s-1} = \sum_{k,l,m,i,j=1}^{3}\sum_{n_k,n_{ij},n_{lm}} V_{1|n_k n_{ij} n_{lm}}\big(l_0^k\big)^{n_k}\big(l_1^{+ij}\big)^{n_{ij}}\big(l_{11}^{+lm}\big)^{n_{lm}}|0\rangle\otimes|0\rangle\otimes|0\rangle \quad (48)$$

with dimensionless coupling constant g, with integers n_k, n_{ij}, n_{lm} satisfying to the requirement of dimensionless of the action and to spin the condition: $n_{ij}+2n_{lm} = 3s-1$. We imply the same representations with unknown coefficients $V_{n_k n_{ij} n_{lm}}$, $F_{n_k n_{ij} n_{lm}}$ with respective restrictions on the integers for the vertex $|V\rangle_{3s}$ and for the structure constant $|F\rangle_{3(s-1)}$.

A consistent deformation of the free BRST-BFV action $\mathcal{S}_{F|s}(\phi)$ leads to the relations

$$\delta_1\mathcal{S}_{F|s}(\phi) + \delta_0\mathcal{S}_{1|s}(\phi) = 0, \quad \big[\delta_{[1],\xi_1},\, \delta_{[1],\xi_2}\big]|\phi\rangle_s = \delta_{[1],\xi_3}|\phi\rangle_s + o(\phi^2),\ \xi_3 = \xi_3(\xi_1,\xi_2). \quad (49)$$

Equivalently, the consistency in the deformation of the classical action is to be controlled by the solvability of the master equation for a deformed BRST–BV action in the language of a component antibracket $(\bullet,\bullet) = \frac{\overleftarrow{\delta}\bullet}{\delta\phi^A_{\min}}\frac{\overrightarrow{\delta}\bullet}{\delta\phi^*_{A\min}} - \frac{\overleftarrow{\delta}\bullet}{\delta\phi^*_{A\min}}\frac{\overrightarrow{\delta}\bullet}{\delta\phi^A_{\min}}$,

$$\big(S_{[1]|s}, S_{[1]|s}\big) = 2\int d^dx\Bigg(\frac{\overleftarrow{\delta}S_{[1]|s}}{\delta\phi^{(\mu)_s}(x)}\frac{\overrightarrow{\delta}S_{[1]|s}}{\delta\phi^*_{(\mu)_s}(x)} + \frac{\overleftarrow{\delta}S_{[1]|s}}{\delta C^{(\mu)_{s-1}}(x)}\frac{\overrightarrow{\delta}S_{[1]|s}}{\delta C^*_{(\mu)_{s-1}}(x)}\Bigg) = 0. \quad (50)$$

A detailed consideration of self-interaction and interaction vertices (involving TS HS fields of different helicities) according to the proposed algorithm poses a separate problem.

Let us now turn ourselves to a non-minimal extension of the BRST–BV approach for the TS HS field in question.

3 Non-minimal BRST-BV Lagrangians

Since all the monomials amongst the minimal BFV ghost oscillators have been already utilized to compose a minimal generalized vector $|\chi^0_{\mathrm{g}|c}\rangle$ (23), we need to enlarge the concept of BFV–BV duality beyond the minimal sector. In the first place, we augment our constrained BRST–BV approach by a Lagrangian $S_{C|s}$ (29) for a field of spin s in the triplet form, by introducing BFV non-minimal oscillators of antighosts $\overline{C}^a$, $\overline{\mathcal{P}}_a$ and Lagrangian multipliers λ^a, π_a according to the numbers $N_{\mathrm{nmin}} = 4(n_{o_a}-1) = 8$ (with no allowance for l_0) in order to present a total constrained BRST operator $Q_{c|tot}$ with the properties (for a vanishing gh_L)

	$\overline{\eta}_1$	$\overline{\eta}_1^+$	$\overline{\mathcal{P}}_1^+$	$\overline{\mathcal{P}}_1$	λ_1	λ_1^+	π_1	π_1^+
ϵ	1	1	1	1	0	0	0	0
gh_H	-1	-1	1	1	0	0	0	0

, (51)

$$\{\overline{\eta}_1,\, \overline{\mathcal{P}}_1^+\} = \{\overline{\eta}_1^+,\, \overline{\mathcal{P}}_1\} = 1,\ \big[\lambda_1,\, \pi_1^+\big] = \big[\pi_1,\, \lambda_1^+\big] = 1. \quad (52)$$

The operator $Q_{c|tot}$ depending on the whole set of BFV oscillators Γ_{gh} required to construct a unitarizing Hamiltonian for a topological dynamical system [11], as well as a total BRST-extended constraint $\mathcal{L}_{11}$ and a spin operator $\sigma_{c|tot}$, which act on a total Hilbert space $\mathcal{H}^{\mathrm{nmin}}_{g|\mathrm{tot}} \equiv \mathcal{H}_{g|\mathrm{tot}} \otimes \mathcal{H}_{\mathrm{nmin}}$ are found as solutions of *non-minimal generating equations* of the form (6):

$$Q_{c|tot} \;=\; Q_c + \overline{\mathcal{P}}_1\pi_1^+ + \pi_1\overline{\mathcal{P}}_1^+, \tag{53}$$

$$\mathcal{L}_{11} \;=\; \widehat{L}_{11} + \overline{\eta}_1\overline{\mathcal{P}}_1 + \lambda_1\pi_1, \quad \sigma_{c|tot} \;=\; \widehat{\sigma}_c + \overline{\eta}_1^+\overline{\mathcal{P}}_1 - \overline{\eta}_1\overline{\mathcal{P}}_1^+ + \lambda_1^+\pi_1 + \lambda_1\pi_1^+. \tag{54}$$

Second, we extend the minimal BRST-BV approach to a non-minimal one by introducing a tensor antighost, a Nakanishi-Lautrup fields and their antifields of the non-minimal BV sector (for a triplet LF), with the respective (ϵ, gh_L) distributions (for $gh_H \equiv 0$)

$$\{\phi^A\} = \bigl\{\phi^A_{\mathrm{min}}; \overline{C}^{(\mu)_{s-1}}(x),\, B^{(\mu)_{s-1}}(x)\bigr\}, \quad \{\phi^*_A\} = \bigl\{\phi^*_{A\,\mathrm{min}}; \overline{C}^*_{(\mu)_{s-1}}(x), B^*_{(\mu)_{s-1}}(x)\bigr\}, \tag{55}$$

	$\overline{C}^{(\mu)_{s-1}}$	$B^{(\mu)_{s-1}}$	$\overline{C}^*_{(\mu)_{s-1}}$	$B^*_{(\mu)_{s-1}}$
ϵ	1	0	0	1
gh_L	-1	0	0	-1

. (56)

These (anti)fields are multiplied (inside the respective monomials of a *total generalized vector* $|\chi^0_{\mathrm{tot}|c}\rangle_s \in \mathcal{H}^{0|\mathrm{nmin}}_{g|\mathrm{tot}}$) only by non-minimal BFV oscillators (where $\mathcal{H}^{0|\mathrm{nmin}}_{g|\mathrm{tot}} \equiv \sum_{e+l=0}\mathcal{H}^{e,l|\mathrm{nmin}}_{g|\mathrm{tot}}$ for $\mathcal{H}^{\mathrm{nmin}}_{g|\mathrm{tot}}$ admitting the natural $\mathbb{Z}\oplus\mathbb{Z}$ grading $\mathcal{H}^{\mathrm{nmin}}_{g|\mathrm{tot}} = \sum_{e,l\geq 0}\mathcal{H}^{e,l|\mathrm{nmin}}_{g|\mathrm{tot}}$), namely,

$$|\chi_{\mathrm{tot}|c}\rangle = \sum_n \frac{\imath^n}{n!}\prod(\eta_0)^{n_{f0}}\prod(\eta_1^+)^{n_f}(\mathcal{P}_1^+)^{n_p}(\overline{\eta}_1^+)^{n_{\bar f}}(\overline{\mathcal{P}}_1^+)^{n_{\bar p}}\prod(\lambda_1^+)^{n_\lambda}(\pi_1^+)^{n_\pi}\phi^{N_{\mathrm{tot}}}_{\mathrm{tot}|c}(a^+)|0\rangle. \tag{57}$$

with a chosen representation $(\overline{\eta}_1, \overline{\mathcal{P}}_1, \lambda_1, \pi_1)|0\rangle = 0$ and $N_{\mathrm{tot}} \equiv (n_{f0}, n_f, n_p, n_{\bar f}, n_{\bar p}, n_\lambda, n_\pi)$.

Once again, the same spectral problem (21), albeit for the $Q_{c|\mathrm{tot}}$-complex with imposed spin, the BRST-extended constraint (54) in $\mathcal{H}^{\mathrm{nmin}}_{g|\mathrm{tot}}$ leads to the representation

$$\begin{aligned} |\chi^0_{\mathrm{tot}|c}\rangle_s &= |\chi^0_{\mathrm{g}|c}\rangle_s + \overline{\mathcal{P}}_1^+\big|\overline{C}(a^+)\rangle_{s-1} + \lambda_1^+\big|b(a^+)\rangle_{s-1} + \eta_0\bigl(\overline{\eta}_1^+\big|\overline{C}^*(a^+)\rangle_{s-1} + \pi_1^+\big|b^*(a^+)\rangle_{s-1}\bigr) \\ &\equiv |\chi^0_{\mathrm{g}|c}\rangle_s + \big|\overline{C}(\overline{\mathcal{P}}_1^+, a^+)\rangle_s + \big|b(\lambda_1^+, a^+)\rangle_s + \big|\overline{C}^*(\overline{\eta}_1^+, a^+)\rangle_s + \big|b^*(\pi_1^+, a^+)\rangle_s \end{aligned} \tag{58}$$

$[(\epsilon, gh_{tot})|\chi^0_{\mathrm{tot}|c}\rangle = (0,0)]$. Here, we have used the rule (25) augmented in the non-minimal sector to construct antifield vectors for $\big|\overline{C}(\overline{\mathcal{P}}_1^+, a^+)\rangle$ and $\big|b(\lambda_1^+, a^+)\rangle_s$. As a result, the Grassmann-even functional

$$S_{0|s}(\chi^{0|\mathrm{nmin}}_{\mathrm{tot}|c}) \;=\; \int d\eta_0\; {}_s\langle\chi^{0|\mathrm{nmin}}_{\mathrm{tot}|c}\big|Q_{c|\mathrm{tot}}\big|\chi^{0|\mathrm{nmin}}_{\mathrm{tot}|c}\rangle_s, \quad \mathcal{L}_{11}\big|\chi^{0|\mathrm{nmin}}_{\mathrm{tot}|c}\rangle_s = 0 \tag{59}$$

is nothing else than the BV action $S_{0|s} = S_{ext}(\phi, \phi^*)$ in the constrained formulation with allowance for the fact that $(\mathcal{L}_{11}, \sigma_{c|tot})|\chi_{\mathrm{g}|c}\rangle = (\widehat{L}_{11}, \widehat{\sigma}_c)|\chi_{\mathrm{g}|c}\rangle$. The action $S_{0|s}$ in the η_0-independent, ghost-independent, tensor triplet representations acquires the forms

$$S_{0|s} = S_{C|s} + \int d\eta_0\, \Bigl\{{}_s\langle\overline{C}^*(\overline{\eta}_1^+, a^+)\big|\bigl(\overline{\mathcal{P}}_1\pi_1^+ + \pi_1\overline{\mathcal{P}}_1^+\bigr)|b(\lambda_1^+, a^+)\rangle_s + h.c.\Bigr\} \tag{60}$$

$$= S_{C|s} - {}_{s-1}\langle\overline{C}^*(a^+)\big|b(a^+)\rangle_{s-1} - {}_{s-1}\langle b(a^+)\big|\overline{C}^*(,a^+)\rangle_{s-1} \tag{61}$$

$$= S_{C|s} + 2\frac{(-1)^s}{s!}\int d^dx\; s\overline{C}^*_{(\mu)_{s-1}}b^{(\mu)_{s-1}} \tag{62}$$

with traceless tensor (anti)fields from non-minimal sector (for $\mathbb{R}$-valued $\overline{C}^*_{(\mu)_{s-1}}$, $b^{(\mu)_{s-1}}$ and pure imaginary $\big(\overline{C}^{(\mu)_{s-1}},\, b^*_{(\mu)_{s-1}}\big)^+ = -\big(\overline{C}^{(\mu)_{s-1}},\, b^*_{(\mu)_{s-1}}\big)\big)$.

Notice that the doublet and single-field forms of the BV actions are determined by the functionals $S^d_{C|s}$ (33), $S_{F|s}$ (34), with the same non-minimal extension $\overline{C}^*b$. The functional $S_{0|s}$ satisfies the master equation (50) with an appropriate antibracket written for the respective (triplet, doublet or single-field) representation in the total field-antifield space. It is invariant with respect to the Lagrangian BRST transformations of the field vector $|\chi^0_{\mathrm{f}|c}\rangle$, when presenting $|\chi^0_{\mathrm{tot}|c}\rangle = |\chi^0_{\mathrm{f}|c}\rangle + |\chi^0_{\mathrm{af}|c}\rangle$ for $|\chi^0_{(\mathrm{a})\mathrm{f}|c}\rangle$ depending on (anti)fields for ket-vector and its dual:

$$\delta_B|\chi^0_{\mathrm{f}|c}(x)\rangle_s = \mu\frac{\overrightarrow{\delta}}{\delta_s\langle\chi^0_{\mathrm{af}|c}(x)\big|}S_{0|s} \;=\; \mu\Big(Q_c|C(x)\rangle_s + \big[\overline{\mathcal{P}}_1\pi_1^+ + \pi_1\overline{\mathcal{P}}_1^+\big]\big|b(\lambda_1^+,a^+,x)\rangle_s\Big), \quad (63)$$

$$\delta_B S_{0|s} = \Big(\delta_{B\,s}\langle\chi^0_{\mathrm{f}|c}|\frac{\overrightarrow{\delta}\,S_{0|s}}{\delta_s\langle\chi^0_{\mathrm{f}|c}\big|} + \frac{S_{0|s}\overleftarrow{\delta}}{\delta|\chi^0_{\mathrm{f}|c}\rangle_s\big|}\delta_B|\chi^0_{\mathrm{f}|c}\rangle_s\Big) = \delta_B S_{C|s} + \delta_B\big(S_{0|s} - S_{C|s}\big) = 0. \quad (64)$$

In a ghost-independent form, the BRST transformations take the usual form, as we omit the symbol "s"

$$\delta_B\Big[\,\big(|\phi\rangle, |\phi_1\rangle, |\phi_2\rangle\big)\,, \big|C(a^+)\rangle, \big|\overline{C}(a^+)\rangle, \big|b(a^+)\rangle\Big] = \Big[\,\big(l_1^+, l_0, l_1\big)\,\big|C(a^+)\rangle, 0, \big|b(a^+)\rangle, 0\Big]\mu. \quad (65)$$

4 Gauge-fixing and BRST-invariant quantum action

To determine a non-renormalized quantum action $S_0^\Psi(\chi^{\mathrm{tot}0}_{g|c})$, we introduce a quadratic gauge-fermion functional $\Psi(\chi^0_{\mathrm{tot}|c})$ corresponding to $R_{\xi,\beta}$-gauges with the help of Grassman-even x-local kernel $\widehat{E}^\Psi_{\xi,\beta}$ constructed from BRST-BFV operator gauge-fermion $\Psi_H(\overline{\eta}_1^{(+)}, \pi_1^{(+)}, o_a; \xi, \beta)$:

$$\Psi\left(\chi^0_{\mathrm{tot}|c}\right) \;=\; \int d\eta_0\; {}_s\langle\chi^0_{\mathrm{tot}|c}|\widehat{E}^\Psi_{\xi,\beta}|\chi^0_{\mathrm{tot}|c}\rangle_s, \quad \text{for}\quad \widehat{E}^\Psi_{\xi,\beta} \overset{def}{=} \eta_0\Psi_H{=}\eta_0\Psi_H(o_a,\Gamma_{gh};\xi,\beta), \qquad (66)$$

$$\Psi_H \;=\; \overline{\eta}_1^+\left(l_1 + \mathcal{P}_1\eta_1\Big[(1+\beta)l_1^+ + \frac{2\beta}{2s-4+d}l_1 l_{11}^+\Big] + \frac{\xi}{2}\pi_1\right) - h.c. \equiv \Psi_H^0 - \big(\Psi_H^0\big)^+, \quad (67)$$

with $(\epsilon, gh_H, gh_L, gh_{\mathrm{tot}})\Psi_H = (1,-1,0,-1)$ and $l_{11}^+ = (1/2)a^+_\mu a^{+\mu}$. The property of anti-hermitian conjugation $\Psi_H^+ = -\Psi_H$ provides the hermiticity of $\widehat{E}^\Psi_{\xi,\beta}$: $\big(\widehat{E}^\Psi_{\xi,\beta}\big)^+ = \widehat{E}^\Psi_{\xi,\beta}$. The definition of Ψ_H as $\Psi_H = \overline{C}^a\chi_a$ (67) determines the first-order operator $\widehat{\chi}_a \equiv \widehat{\chi}$ and $\widehat{\chi}^+$ of the gauge condition

$$\widehat{\chi}\big(\big|\chi^0_c\rangle_s + \big|b(\lambda_1^+)\rangle_s\big) = 0, \quad \big({}_s\langle\chi^0_c\big| + {}_s\langle b(\lambda_1)\big|\big)\widehat{\chi}^+ = 0 \qquad (68)$$

being equal in number with that of the independent gauge parameters $|\chi^1_c\rangle_s$ (9). The gauge condition (68), as expanded in ghost powers, is equivalent to three equations

$$l_1|\phi\rangle_s - \Big[(1+\beta)l_1^+ + \frac{2\beta}{2s-4+d}l_1 l_{11}^+\Big]|\phi_2\rangle_{s-2} + \frac{\xi}{2}|b\rangle_{s-1} = 0; \quad l_1|\phi_k\rangle_{s-k} = 0, k=1,2. \quad (69)$$

where the equation for $|\phi_1\rangle_{s-1}$ does not contribute to $\Psi(\chi^0_{\mathrm{tot}|c})$ due to $\eta_0^2 \equiv 0$. For $(\xi,\beta) = 0$ and $(\xi,\beta) = (1,0)$, the gauge (68) corresponds to the Landau and Feynman gauges (for

$s=1$), respectively. Any of the gauge conditions in (69) respects the property of tracelessness (which means $l_{11}\big(|\phi\rangle_s - ...\big) = 0$ and $l_{11}l_1|\phi_k\rangle = 0$ on the traceless constraint surface). In the ghost dependent form it is equivalent to the relation (with allowance for $[l_{11},\, l^+_{11}]|\phi_k\rangle_{s-k} = \{g_0 - (s-k+d/2)\}|\phi_k\rangle_{s-k} = 0$)

$$[\widehat{\chi},\, \mathcal{L}_{11}\}\big(\big|\chi^0_c\rangle_s + \big|b(\lambda^+_1)\rangle_s\big) = \mathcal{P}_1\eta_1 l_1\big|\chi^0_c\rangle_s \overset{l_1|\phi_2\rangle=0}{=} 0. \tag{70}$$

In the ghost-independent representation, the gauge-fermion functional $\Psi(\chi^0_{\mathrm{tot}|c})$ reads (we omit the spin index)

$$\Psi\big(\phi_k, \overline{C}, b\big) = \Big\{\langle\overline{C}(a)\big|\Big(l_1\big|\phi\rangle - \big[(1+\beta)l^+_1 + \frac{2\beta}{2s-4+d}l_1 l^+_{11}\big]\big|\phi_2\rangle + \frac{\xi}{2}\big|b(a^+)\rangle\Big) - h.c.\Big\}. \tag{71}$$

In the single-field (Fronsdal) formulation, the functional (71) transforms as $\Psi_{F|s} \equiv \Psi\big(\phi, \overline{C}, b\big) = \Psi\big(\phi_k, \overline{C}, b\big)|_{|\phi_2\rangle = -l_{11}|\phi\rangle)}$ and reads in the tensor form as

$$\Psi_{F|s} = -2\frac{(-1)^s}{s!}\int d^dx\; s\overline{C}^{(\mu)_{s-1}}\Big\{\partial^{\mu_s}\phi_{(\mu)_s} + (s-1)(1+\beta)\partial_{\mu_{s-1}}\phi_{(\mu)_{s-2}}{}^{\nu}{}_{\nu} \tag{72}$$
$$+ \frac{2\beta}{2s-4+d}(s-1)\big[\partial_{\mu_{s-1}}\phi_{(\mu)_{s-2}}{}^{\nu}{}_{\nu} + \frac{1}{2}(s-2)\eta_{\mu_{s-1}\mu_{s-2}}\partial^{\rho}\phi_{(\mu)_{s-3}\rho}{}^{\nu}{}_{\nu}\big] + \frac{\xi}{2}b_{(\mu)_{s-1}}\Big\}.$$

The ghost-dependent Grassmann-odd density $\mathcal{M}_{\Psi|c}(x,y) \equiv \mathcal{M}_{\Psi|c}(\Gamma_{gh}, a^{(+)}, x, y)$ of the Faddeev-Popov operator $M_{\Psi|c}(x,y)$, which is implied by a variation of the gauge condition (in terms of Ψ^0_H) under the gauge transformation $\delta\big(\Psi^0_H\big|\chi^0_c(x)\rangle\big)$ (in order to extract a single representative from a gauge orbit) admits the representation

$$\mathcal{M}_{\Psi|c}(x,y) \overset{def}{=} \eta_0\int d^dy\Big\{\Psi^0_H\big|\chi^0_c(x)\rangle_s\frac{\overleftarrow{\delta}_{\eta_0}}{\delta\big|\chi^0_c(y)\rangle_s}Q_c(y) - Q_c(y)\frac{\overrightarrow{\delta}_{\eta_0}}{\delta_s\langle\chi^0_c(y)\big|}\Big({}_s\langle\chi^0_c(x)\big|(\Psi^0_H)^+\Big)\Big\} \tag{73}$$
$$= \eta_0\int d^dy\Big(\Psi^0_H|_{\xi=0}(x)\delta(x,y)Q_c(y) - Q_c(y)(\Psi^0_H)^+|_{\xi=0}(x)\delta(x,y)\Big),$$
$$\Longrightarrow \int d^dy\mathcal{M}_{\Psi|c}(x,y)\big|\chi^1_c(y)\rangle_s = \eta_0\overline{\eta}^+_1\eta_1\Big(l_0 - \beta l^+_1 l_1 - \frac{2\beta}{2s-4+d}l_1 l^+_{11} l_1\Big)\big|\chi^1_c(x)\rangle_s = 0 \tag{74}$$

where the following values are the only vanishing ones: $\big|\chi^1_c(x)\rangle_s = \mathcal{P}^+_1\big|\xi(a^+,y)\rangle_{s-1}$, $(gh_H, gh_{tot})\big|\chi^1_c(x)\rangle = (-1,-1)$. The non-minimal Faddeev-Popov operator $M_{\Psi|c}(x,y)$, (for $\beta\neq 0$ and $\mathcal{P}^+_1\overline{\mathcal{P}}_1\mathcal{P}_0\mathcal{M}_{\Psi|c}(x,y) = \imath M_{\Psi|c}(x,y)$) acquires the tensor form

$$\int d^dy M_{\Psi|c}(a^{(+)}; x,y)\big|\xi(a^+,y)\rangle_{s-1} = \Big(l_0 - \beta l^+_1 l_1 - \frac{2\beta}{2s-4+d}l_1 l^+_{11} l_1\Big)\big|\xi(a^+,x)\rangle_{s-1}, \tag{75}$$

$$M_{\Psi|c}{}^{(\nu)_{s-1}}_{(\mu)_{s-1}}(x) \;=\; \partial^2\delta^{(\nu)_{s-1}}_{(\mu)_{s-1}} + K^{(\nu)_{s-1}}_{(\mu)_{s-1}}\big(\partial,\beta\big) \text{ from } \big(M_{\Psi|c}(x)\xi(x)\big)^{(\mu)_{s-1}}\prod_{i=1}^{s-1}(a^+)_{(\mu)_i}|0\rangle, \tag{76}$$

$$K^{(\nu)_{s-1}}_{(\mu)_{s-1}}\big(\partial,\beta\big) = \frac{\beta}{2s-4+d}\eta^{\{\nu_{s-2}\nu_{s-1}}\partial_{\mu_{s-1}}\partial_{\mu_{s-2}}\delta^{(\nu)_{s-3}\}}_{(\mu)_{s-3}} - \beta\frac{s-3+(d/2)}{s-2+(d/2)}\partial_{\mu_{s-1}}\partial^{\{\nu_{s-1}}\delta^{(\nu)_{s-2}\}}_{(\mu)_{s-2}},$$

$$\text{for } \delta^{(\nu)_{s-k}}_{(\mu)_{s-k}} = \delta^{\{\nu_1}_{\mu_1}\ldots\delta^{\nu_{s-k}\}}_{\mu_{s-k}} = \frac{1}{(s-k)!}\Big[\delta^{\nu_1}_{\mu_1}\ldots\delta^{\nu_{s-k}}_{\mu_{s-k}} + cycl.perm.(\nu_1,...,\nu_{s-k})\Big] \tag{77}$$

Let us note, in the first place, that we have omitted in (74) the term $\overline{\eta}^+_1\eta^+_1(l_1)^2$ and also its dual $\eta_1\overline{\eta}_1(l^+_1)^2$, since it vanishes as one estimates the scalar product for the ghost-antighost term $\int d\eta_{0s}\bigl(\langle\overline{C}\bigl|\mathcal{M}_{\Psi|c}(x,y)\bigr|C\rangle_s + h.c.\bigr)$. Secondly, the variational derivatives for a fixed η_0, $\overleftarrow{\delta}_{\eta_0}/\delta\bigl|\chi^0_c(y)\rangle_s$, $\overrightarrow{\delta}_{\eta_0}/\delta_s\langle\chi^0_c(y)\bigr|$ are calculated in accordance with the rules (41) of the superfield BRST–BV quantization [50, 51].

We can now determine the quantum BRST–BV action $S^{\Psi}_{0|s} \equiv S^{\Psi}_{0|s}\bigl(\chi^0_{\mathrm{tot}|c}\bigr)$ as a shift of the vector $\bigl|\chi^0_{\mathrm{tot}|c}\rangle_s$ by a variational derivative of the gauge-fermion functional:

$$\bigl|\chi^0_{\mathrm{tot}|c}\rangle_s \to \bigl|\chi^{\Psi 0}_{\mathrm{tot}|c}\rangle_s = \bigl|\chi^0_{\mathrm{tot}|c}\rangle_s + \frac{\overrightarrow{\delta}}{\delta_s\langle\chi^0_{\mathrm{tot}|c}\bigr|}\Psi = \Bigl\{1+\eta_0\frac{\overrightarrow{\delta}_{\eta_0}}{\delta_s\langle\chi^0_{\mathrm{tot}|c}\bigr|}{}_s\langle\chi^0_{\mathrm{tot}|c}\bigr|\Psi_H\Bigr\}\bigl|\chi^0_{\mathrm{tot}|c}\rangle_s \quad (78)$$

$$\Longrightarrow \bigl(|\chi^{\Psi 0}_{\mathrm{f}|c}\rangle_s,\, |\chi^{\Psi 0}_{\mathrm{af}|c}\rangle_s\bigr) \;=\; \bigl(|\chi^0_{\mathrm{f}|c}\rangle_s,\, |\chi^0_{\mathrm{af}|c}\rangle_s + \eta_0\Psi_H\bigl|\chi^0_{\mathrm{f}|c}\rangle_s\bigr), \quad (79)$$

where the antifield components are the only ones that change under the notation $\Delta_\Psi|A\rangle \equiv |A^\Psi\rangle - |A\rangle$,

$$\Delta_\Psi|\chi^{0*}_c\rangle_s = -\eta_0\widehat{\chi}^+\bigl|\overline{C}(a^+)\rangle_{s-1}, \qquad \Delta_\Psi\bigl|\overline{C}^*(\overline{\eta}^+_1, a^+)\rangle_s = -\eta_0\overline{\eta}^+_1\widehat{\chi}\bigl(\bigl|\chi^0_c\rangle_s + \bigl|b(\lambda^+_1)\rangle_s\bigr), \quad (80)$$

$$\Delta_\Psi|C^*\rangle_s = 0, \qquad \Delta_\Psi|b^*(\pi^+_1, a^+)\rangle_s = -(\xi/2)\eta_0\pi^+_1\bigl|\overline{C}(a^+)\rangle_{s-1}. \quad (81)$$

Extended by antifields and usual $\mathcal{S}^{\Psi}_{0|s}$ (for $|\chi^0_{\mathrm{af}|c}\rangle = 0$) quantum actions read

$$S^{\Psi}_{0|s} = S_{0|s}\bigl(\chi^{\Psi 0}_{\mathrm{tot}|c}\bigr) = S_{0|s}\bigl(\chi^0_{\mathrm{f}|c}, \chi^{\Psi 0}_{\mathrm{af}|c}\bigr) \;=\; \int d\eta_0\; {}_s\langle\chi^{\Psi 0}_{\mathrm{tot}|c}\bigl|Q_{c|\mathrm{tot}}\bigr|\chi^{\Psi 0}_{\mathrm{tot}|c}\rangle_s, \quad (82)$$

$$\mathcal{S}^{\Psi}_{0|s} = S_{0|s}\bigl(\widetilde{\chi}^{\Psi 0}_{\mathrm{tot}|c}\bigr) \;=\; \int d\eta_0\; {}_s\langle\widetilde{\chi}^{\Psi 0}_{\mathrm{tot}|c}\bigl|Q_{c|\mathrm{tot}}\bigr|\widetilde{\chi}^{\Psi 0}_{\mathrm{tot}|c}\rangle_s, \text{ for } \bigl|\widetilde{\chi}^{\Psi 0}_{\mathrm{tot}|c}\rangle \equiv \bigl|\chi^{\Psi 0}_{\mathrm{tot}|c}\rangle|_{(|\chi^0_{\mathrm{af}|c}\rangle=0)}. \quad (83)$$

By construction, the functional $S^{\Psi}_{0|s}$ presented in the η_0-independent, ghost-independent and tensor forms

$$S^{\Psi}_{0|s} = \mathcal{S}_{C|s} + \Biggl(\bigl({}_s\langle S^*_c\bigr|, {}_s\langle B^*_c\bigr| - {}_{s-1}\langle\overline{C}\bigr|\widehat{\chi}\bigr)\begin{pmatrix} l_0 & -\Delta Q_c \\ -\Delta Q_c & \eta^+_1\eta_1\end{pmatrix}\begin{pmatrix}\bigl|C\rangle_s \\ 0\end{pmatrix} \quad (84)$$

$$- \Bigl({}_s\langle\overline{C}^*(\overline{\eta}_1, a)\bigr| - \bigl({}_s\langle\chi^0_c\bigr| + {}_s\langle b(\lambda_1)|\bigr)\widehat{\chi}^+\overline{\eta}_1\Bigr)\overline{\mathcal{P}}^+_1\pi_1|b(\lambda^+_1, a^+)\rangle_s + h.c.\Biggr)$$

$$= S_{0|s} + \Bigl({}_{s-1}\langle\overline{C}\bigl|\widehat{\chi}l^+_1|C\rangle_{s-1} + \bigl({}_s\langle\chi^0_c\bigr| + {}_s\langle b(\lambda_1)|\bigr)\widehat{\chi}^+|b(a^+)\rangle_{s-1} + h.c\Bigr) \quad (85)$$

$$= S_{0|s} + 2\frac{(-1)^s}{s!}\int d^dx\; s\Bigl\{\bigl(\overline{C}M_{\Psi|c}C\bigr)_{s-1} + \Bigl(\partial^{\mu_s}\phi_{(\mu)_s} + 2(s-1)\Bigl[(1+\beta)\partial_{\mu_s-1}\delta^{\rho}_{\mu_{s-2}}$$

$$+ \frac{2\beta}{2s-4+d}\bigl[2\partial_{\mu_s-1}\delta^{\rho}_{\mu_{s-2}} + (s-2)\eta_{\mu_{s-1}\mu_{s-2}}\partial^{\rho}\bigr]\Bigr]\phi_{2(\mu)_{s-3}\rho} + \frac{\xi}{2}b_{(\mu)_{s-1}}\Bigr)b^{(\mu)_{s-1}}\Bigr\} \quad (86)$$

satisfies, once again, the master equation (50) in the total field-antifield space with the respective antibracket. Note, that we have used in (86) a notation for the ghost Faddeev-Popov term $\bigl(\overline{C}M_{\Psi|c}C\bigr)_{s-1} \equiv \overline{C}_{(\nu)_{s-1}}M_{\Psi|c}{}^{(\nu)_{s-1}}_{(\mu)_{s-1}}(x)C^{(\mu)_{s-1}}$. The quantum extended actions $S^{\Psi}_{F|s}$, for instance, in the single-field (Fronsdal) form for the tensor representation are obtained from (86) by setting $\phi^*_1 = \phi^*_2 = 0$ and expressing ϕ_1 from the algebraic equation of motion in terms of ϕ, ϕ_2, as well as $\phi_2^{(\mu)_{s-2}} = (1/2)\phi^{(\mu)_{s-2}\mu}{}_{\mu}$. The quantum action $\mathcal{S}^{\Psi}_{0|s}$ has a standard structure composed by the classical, ghost and gauge-fixed parts, $\mathcal{S}^{\Psi}_{0|s} = \mathcal{S}_0 + S_{\mathrm{gh}} + S_{\mathrm{gf}}$, and coincides with the known quantum action in the tensor form, modulo the common factor $2(-1)^s/s!$. The actions $S^{\Psi}_{0|s}$ and $\mathcal{S}^{\Psi}_{0|s}$ are both invariant under the BRST transformations

(63) and are also non-degenerate in the total configuration space of ϕ^A, thus providing a definition for the extended ($Z_0\left[J^0,\phi^*\right]$) and usual ($\chi^0_{\mathrm{af}|c}=0$) generating functionals of Green's functions, with a new *generalized vector of external sources* ${}_s\langle J^0_{\mathrm{f}|c}\big|$ ($\big|J^0_{\mathrm{f}|c}\rangle_s$)

$$
{}_s\langle J^0_{\mathrm{f}|c}\big| = {}_s\langle J^0_{c}\big| + \langle 0|J^C_{s-1}(a)\eta_1\eta_0 + \langle J^{\overline{C}}_{s-1}(a)\big|\overline{\eta}_1\eta_0 + \langle 0|J^b_{s-1}(a)\lambda_1\eta_0 \tag{87}
$$

$$
{}_s\langle J^0_{c}\big| = \Big({}_s\langle J(a)\big| + {}_{s-2}\langle J_2(a)\big|\eta_1\mathcal{P}_1\Big)\eta_0 + {}_{s-1}\langle J_1(a)\big|\eta_1, \quad \text{for} \quad (\epsilon, gh_{tot})\langle J^0_{\mathrm{f}|c}\big| = (1,1) \tag{88}
$$

to the generalized field vector $\big|\chi^0_{\mathrm{f}|c}\rangle_s$ (${}_s\langle\chi^0_{\mathrm{f}|c}\big|$). These vectors (with appropriate complex conjugation rules for its tensor components) contains the usual sources $|J_k\rangle_{s-k}$, (${}_{s-k}\langle J_k|$) for the field vectors ${}_{s-k}\langle\phi_k|$ ($|\phi_k\rangle_{s-k}$) for $k=0,1,2$. We determine the functional $Z_0\left[J^0,\phi^*\right]$ in the form

$$
Z_0\left[J^0,\phi^*\right] \;=\; \int d\chi^0_{\mathrm{f}|c}\exp\left\{\frac{\imath}{\hbar}\left[S^{\Psi}_{0|s}\big(\chi^0_{\mathrm{tot}|c}\big) + \int d\eta_0\Big({}_s\langle J^0_{\mathrm{f}|c}|\chi^0_{\mathrm{f}|c}\rangle_s + {}_s\langle \chi^0_{\mathrm{f}|c}|J^0_{\mathrm{f}|c}\rangle_s\Big)\right]\right\} \tag{89}
$$

with the measure $d\chi^0_{\mathrm{f}|c}=\prod_x d\phi^{(\mu)_s}_(x)d\phi_1^{(\mu)_{s-1}}(x)d\phi_2^{(\mu)p_{s-2}}(x)dC^{(\mu)_{s-1}}(x)d\overline{C}^{(\mu)_{s-1}}(x)db^{(\mu)_{s-1}}(x)$ determined for triplet, doublet [without $d\phi_1$] and for single-field [without $d\phi_1 d\phi_2$] formulations. The respective Green functions can be obtained by differentiation with respect to the external sources, e.g. the 2-point function $G^{(2)}(a^{(+)};x,y)$ for initial TS field $|\phi\rangle_s$ in the single-field form (16) for $J_0(x)\equiv J(x)$, $\xi\neq 0$

$$
G^{(2)}(a^{(+)};x,y) \;=\; \frac{\overrightarrow{\delta}}{\delta_s\langle J(x)|}Z_0\left[J^0,\phi^*\right]\frac{\overleftarrow{\delta}}{\delta|J(y)\rangle_s}\Big|_{J^0_c=\phi^*=0} \tag{90}
$$

$$
= \left[l_0 - l_1^+l_1 - (l_1^+)^2l_{11} - l_{11}^+l_1^2 - l_{11}^+(l_0+l_1l_1^+)l_{11} + \xi^{-1}\widehat{\chi}_0^+\widehat{\chi}_0\right]^{-1}\delta(x,y),
$$

$$
\widehat{\chi}_0 \;=\; \widehat{\chi}|_{\xi=0} = l_1 + \left[(1+\beta)l_1^+ + \frac{2\beta}{2s-4+d}l_1l_{11}^+\right]l_{11} \tag{91}
$$

using the equations (68), (69). In the Feynman gauge, the Green function takes the minimal form, $G^{(2)}_{(\xi,\beta)=(1,0)} = (l_0 - l_{11}^+l_0l_{11})^{-1}\delta(x,y)$. Due to the BRST transformations (63) for the integrand in $Z_0^{\Psi}=Z_0\left[0,\phi^*\right]$, the Ward Identity for Z_0 can be presented as follows:

$$
\int \eta_0\Big({}_s\langle J^0_{\mathrm{f}|c}|\frac{\overrightarrow{\delta}}{\delta_s\langle\chi^0_{\mathrm{af}|c}|}Z_0\left[J^0,\phi^*\right] + Z_0\left[J^0,\phi^*\right]\frac{\overleftarrow{\delta}}{\delta|\chi^0_{\mathrm{af}|c}\rangle_s}|J^0_{\mathrm{f}|c}\rangle_s\Big) = 0. \tag{92}
$$

The gauge independence of Z_0^{Ψ}: $Z_0^{\Psi}=Z_0^{\Psi+\delta\Psi}$ upon an admissible change of the gauge condition $\Psi\to\Psi+\delta\Psi$ (e.g., by varying $(\xi,\beta)\to(\xi+\delta\xi,\beta+\delta\beta)$) can be easily established. Inserting, instead of the quadratic action $S^{\Psi}_{0|s}$, the action of an interacting model $S^{\Psi}_{[1]|s}$ constructed according to the recipe (43) with shifted antifields (79), we obtain a non-trivial generating functional $Z\left[J^0,\phi^*\right]$ of Green's functions in the BRST-BV formalism which leads to a non-trivial S-matrix.

5 Conclusion

We have extended the constrained BRST-BFV and BRST-BV methods for constructing the irreducible gauge-invariant Lagrangian $\mathcal{S}_{C|s}=\int d\eta_{0s}\langle\chi^0_c|Q_c|\chi^0_c\rangle_s$ (3) and minimal BRST-BV

$S_{C|s} = \int d\eta_0\ {}_s\langle\chi^0_{\mathrm{g}|c}|Q_c|\chi^0_{\mathrm{g}|c}\rangle_s$ (29) actions for massless totally-symmetric tensor field of helicity s up to the non-minimal BRST–BV method in order to obtain the quantum BV action, ,$\mathcal{S}^{\Psi}_{0|s} = \int d\eta_0\ {}_s\langle\widetilde{\chi}^{\Psi 0}_{\mathrm{tot}|c}|Q_{c|\mathrm{tot}}|\widetilde{\chi}^{\Psi 0}_{\mathrm{tot}|c}\rangle_s$ (83) and the generating functional of Green's functions (89) explicitly in terms of the appropriate Fock space vectors. These vectors contain as their component functions the whole set of the fields in the respective triplet, doublet and single-field formulations, together with the ghost, antighost and Nakanishi-Lautrup fields and respective external sources, with the minimal Hamiltonian BFV ghost oscillators augmented by additional four oscillator pairs $\overline{\eta}_1, \overline{\mathcal{P}}^+_1$; $\overline{\eta}^+_1, \overline{\mathcal{P}}_1$; λ_1, π^+_1; λ^+_1, π_1 (51) from the non-minimal sector. The latter operators are shown to be necessary, in the first place, for augmenting the generalized vector $|\chi^0_{\mathrm{g}|c}\rangle_s$, the BRST operator Q_c, the BRST-extended constraint $\widehat{L}_{11}$ and the spin operator $\widehat{\sigma}_c$ up to the respective total quantities $\big|\chi^0_{\mathrm{tot}|c}\rangle_s$, $Q_{c|\mathrm{tot}}$, $\mathcal{L}_{11}$ and $\sigma_{c|\mathrm{tot}}$. Secondly, they are used to formulate a Lagrangian gauge-fixing fermion functional $\Psi(\chi^0_{\mathrm{tot}|c})$ (66), with the help of a Hamiltonian operator gauge-fixing fermion Ψ_H (67), in fact as its kernel. The gauge-fixing fermion corresponds to the 2-parametric family of the gauges which extends the case .of R_ξ-gauges. The non-minimal BV action and the gauge-fixed quantum action, obtained using a shift of the antifield components in the total generalized vector $\big|\chi^0_{\mathrm{tot}|c}\rangle_s$ by a variational derivative of the gauge-fermion functional, $\big|\widetilde{\chi}^{\Psi 0}_{\mathrm{tot}|c}\rangle_s$ (78), (79), satisfy a master equation and are (along with the integrand of the vacuum generating functional) invariant under the Lagrangian BRST transformations (63).

We have obtained the various representations for the quantum action in the so called η_0-independent, ghost-independent and tensor forms both for triplet and doublet as well as for single-field (Fronsdal) formulations, which are naturally derived from the BRST-BV quantum action $\mathcal{S}^{\Psi}_{0|s}$.

The proposed non-minimal BRST–BV approach to constructing a quantum action for free and interacting massless TS HS fields allows one to formulate the Feynman quantization rules explicitly in terms of a generating functional of Green's functions $Z_0\left[J^0, \phi^*\right]$ determined using a generalized vector of external sources, and also to finalize the concept of BFV–BV duality between Hamiltonian and Lagrangian quantities in a way different from that suggested in [51].

Remarkably, all the ingredients required for conventional quantization according to perturbation theory – as regards establishing the gauge-independence of the vacuum functional from the choice of admissible gauge conditions, as well as deriving the Ward identity and formulating the Green functions and the Faddeev–Popov operator in a manifest form – can be provided using operations with Fock-space vectors, as has been done earlier in the superfield Lagrangian quantization [50], [51].

There are many directions for applications and development of the suggested approach, such as the quantum action and Feynman rules for a constrained TS fields of helicity s in anti-de-Sitter backgrounds, as well as for unconstrained TS integer HS fields in Minkowski spaces.

Acknowledgements The authors are thank to I.L. Buchbinder, P.Yu. Moshin, M. Najafizadeh for the comments and for valuable discussions. A.A.R. gratefully acknowledges the comments made by the participants of Conferences QFTG'2016 and QFTG'2018 where some of the results presented in this research have been announced. The work of A.A.R. has been supported by the Program of Fundamental Research under the Russian Academy of Sciences, 2013–2020.

References

[1] A. Sagnotti, M. Tsulaia, On higher spins and the tensionless limit of string theory, Nucl. Phys. B 682 (2004) 83, [hep-th/0311257].

[2] G.K. Savvidy, Tensionless strings: Physical Fock space and higher spin fields, Int. J. Mod. Phys. A 19 (2004) 3171 [hep-th/0310085].

[3] J. Mourad, Continuous spin particles from a tensionless string theory, AIP Conf. Proc. 861 (2006) 436 [hep-th/0504118].

[4] A. Font, F. Quevedo, S. Theisen, A Comment on Continuous Spin Representations of the Poincaré Group and Perturbative String Theory, Progress of Physics 62 (2014) 975, [arXiv:1302.4771[hep-th]].

[5] M.A. Vasiliev, Higher Spin Gauge Theories in Various Dimensions, Fortsch. Phys. 52 (2004) 702, [hep-th/0401177].

[6] M.A. Vasiliev, Higher-Spin Theory and Space-Time Metamorphoses, Lect. Notes Phys. 892 (2015) 227, [arXiv:1404.1948 [hep-th]].

[7] A. Fotopoulos, M. Tsulaia, Gauge Invariant Lagrangians for Free and Interacting Higher Spin Fields. A Review of the BRST formulation, Int. J. Mod. Phys. A 24 (2008) 1, [arXiv:0805.1346[hep-th]].

[8] M.A. Vasiliev, From Coxeter Higher-Spin Theories to Strings and Tensor Models, JHEP 1808 (2018) 051, [arXiv:1804.06520 [hep-th]].

[9] A.K.H. Bengtsson, A unified action for higher spin gauge bosons from covariant string theory, Phys.Lett. B 182 (1986) 321.

[10] S. Ouvry, J. Stern, Gauge Fields of Any Spin and Symmetry, Phys.Lett. B177 (1986) 335; W. Siegel, B. Zwiebach, Gauge String Fields from the Light Cone Nucl. Phys. B 282 (1987) 125; W. Siegel, Gauging Ramond String Fields Via OSP(1,1/2), Nucl. Phys. B284 (1987) 632.

[11] E.S. Fradkin, G.A. Vilkovisky, Quantization of relativistic systems with constraints, Phys. Lett. B55 (1975) 224; I.A. Batalin, G.A. Vilkovisky, Relativistic S-matrix of dynamical systems with boson and fermion constraints, Phys.Lett. B69 (1977) 309; M. Henneaux, Hamiltonian form of the path integral for theories with a gauge freedom, Phys. Reports 126 (1985) 1.

[12] I.A. Batalin, E.S. Fradkin, Operator Quantization of Relativistic Dynamical Systems Subject to First Class Constraints, Phys. Lett. B128 (1983) 303.

[13] A. Pashnev, M. Tsulaia, Description of the higher massless irreducible integer spins in the BRST approach, Mod. Phys. Lett. A13 (1998) 1853, [arXiv:hep-th/9803207].

[14] C. Burdik, A. Pashnev, M. Tsulaia, Auxiliary representations of Lie algebras and the BRST constructions, Mod. Phys. Lett. A15 (2000) 281, [arXiv:hep-th/0001195].

[15] I.L. Buchbinder, A. Pashnev, M. Tsulaia, Lagrangian formulation of the massless higher integer spin fields in the AdS background, Phys. Lett. B523 (2001) 338, [arXiv:hep-th/0109067]; Massless Higher Spin Fields in the AdS Background and BRST Constructions for Nonlinear Algebras, [arXiv:hep-th/0206026]; X. Bekaert, I.L. Buchbinder, A. Pashnev, M. Tsulaia, On higher spin theory: Strings, BRST, dimensional reductions, Class.Quant.Grav. 21 (2004) S1457, [arXiv:hep-th/0312252].

[16] I.L. Buchbinder, V.A. Krykhtin, A. Pashnev, BRST approach to Lagrangian construction for fermionic massless higher spin fields, Nucl. Phys. B711 (2005) 367, [arXiv:hep-th/0410215]; I.L. Buchbinder, V.A. Krykhtin, L.L. Ryskina, H. Takata, Gauge invariant Lagrangian construction for massive higher spin fermionic fields, Phys.Lett. B 641 (2006) 386, [arXiv:hep-th/0603212].

[17] I.L. Buchbinder, V.A. Krykhtin, BRST approach to higher spin field theories, Nucl. Phys. B 727 (2005) 536, [arXiv:hep-th/0505092].

[18] I.L. Buchbinder, V.A. Krykhtin, A.A. Reshetnyak, BRST approach to Lagrangian construction for fermionic higher spin fields in (A)dS space, Nucl. Phys. B787 (2007) 211, [arXiv:hep-th/0703049]; I.L. Buchbinder, V.A. Krykhtin, P.M. Lavrov, Gauge invariant Lagrangian formulation of higher spin massive bosonic field theory in AdS space, Nucl. Phys. B762 (2007) 344, [arXiv:hep-th/0608005].

[19] C. Burdik, A. Pashnev, M. Tsulaia, On the Mixed symmetry irreducible representations of the Poincare group in the BRST approach, Mod.Phys.Lett. A16 (2001) 731, [arXiv:hep-th/0101201]; The Lagrangian description of representations of the Poincare group, Nucl. Phys. Proc. Suppl. 102 (2001) 285, [arXiv:hep-th/0103143].

[20] A.A. Reshetnyak, P.Yu. Moshin, BRST approach to Lagrangian formulation for mixed-symmpetry fermionic higher-spin fields JHEP 10 (2007) 040, JHEP 05 (2019) 027 (addendum), [arXiv:0707.0386[hep-th]]; Russ. Phys. Journal 56 (2013) 307, [arXiv:1304.7327[hep-th]];
I.L. Buchbinder, V.A. Krykhtin, H. Takata, Gauge invariant Lagrangian construction for massive bosonic mixed symmetry higher spin fields, Phys.Lett. B 856 (2007) 253, [arXiv:0707.2181[hep-th]].

[21] C. Burdik, A. Reshetnyak, On representations of Higher Spin symmetry algebras for mixed-symmetry HS fields on AdS-spaces. Lagrangian formulation, J. Phys. Conf. Ser. 343 (2012) 012102, [arXiv:1111.5516[hep-th]].

[22] I.L. Buchbinder, A.A. Reshetnyak, General Lagrangian Formulation for Higher Spin Fields with Arbitrary Index Symmetry. I. Bosonic fields, Nucl. Phys. B 862 (2012) 270, [arXiv:1110.5044[hep-th]].

[23] A.A. Reshetnyak, General Lagrangian Formulation for Higher Spin Fields with Arbitrary Index Symmetry. 2. Fermionic fields, Nucl. Phys. B 869 (2013) 523, [arXiv:1211.1273[hep-th]].

[24] A.A. Reshetnyak, Gauge-invariant Lagrangians for mixed-antisymmetric higher spin fields, Phys. Part. Nucl. Lett. 14 (2017) 411, [arXiv:1604.00620[hep-th]].

[25] A.A. Reshetnyak, Constrained BRST- BFV Lagrangian formulations for Higher Spin Fields in Minkowski Spaces, JHEP 1809 (2018) 104, [arXiv:1803.04678[hep-th]].

[26] G. Barnich, M. Grigoriev, A. Semikhatov, I. Tipunin,Parent field theory and unfolding in BRST first-quantized terms, Comm. Math. Phys. 260 (2005) 147, [arxiv:hep-th/0406192].

[27] K.B. Alkalaev, M. Grigoriev, I.Yu. Tipunin, Massless Poincare modules and gauge invariant equations, Nucl. Phys. B. 823 (2009) 509, [arXiv:0811.3999[hep-th]].

[28] R.R. Metsaev, BRST-BV approach to cubic interaction vertices for massive and massless higher-spin fields, Phys. Lett. B720 (2013) 237, [arXiv:1205.3131[hep-th]].

[29] I.A. Batalin, G.A. Vilkovisky, Gauge Algebra and Quantization, Phys.Lett. B102 (1981) 27; A Generalized Canonical Formalism and Quantization of Reducible Gauge Theories, Phys.Lett. B120 (1983) 166; Quantization of Gauge Theories with Linearly Dependent Generators, Phys.Rev. D28 (1983) 2567; Phys.Rev.D 30 (1984) 508 (erratum).

[30] A.A. Reshetnyak, Constrained BRST-BFV and BRST-BV Lagrangians for half-integer HS fields on $R^{1,d-1}$, Phys.Part.Nucl. 49 (2018) 952, [arXiv:1803.05173 [hep-th]].

[31] P. Schuster, N. Toro, Continuous-spin particle field theory with helicity correspondence, Phys. Rev. D91 (2015) 025023, [arXiv:1404.0675 [hep-th]].

[32] X. Bekaert, M. Najafizadeh, M.R. Setare, A gauge field theory of fermionic Continuous-Spin Particles, Phys. Lett. B 760 (2016) 320, [arXiv:1506.00973[hep-th]].

[33] R.R. Metsaev, Continuous spin gauge field in (A)dS space, Phys. Lett. B 767 (2017) 458, [arXiv:1610.00657[hep-th]].

[34] I.L. Buchbinder, V.A. Krykhtin, H. Takata, BRST approach to Lagrangian construction for bosonic continuous spin field, Phys.Lett. B 785 (2018) 315, [arXiv:1806.01640[hep-th]].

[35] R.R. Metsaev, Fermionic continuous spin gauge field in (A)dS space, Phys. Lett. B773 (2017) 135, [arXiv:1703.05780[hep-th]].

[36] M. Najafizadeh, Modified Wigner equations and continuous spin gauge field, Phys. Rev. D 97 (2018) 6, 065009, [arXiv:1708.00827[hep-th]].

[37] X. Bekaert, E.D. Skvortsov, Elementary particles with continuous spin, Int. J. Mod. Phys. A32 (2017) 1730019, [arXiv:1708.01030[hep-th]].

[38] M. Khabarov, Yu.M. Zinoviev, Infinite (continuous) spin fields in the frame-like formalism, Nucl. Phys. B928 (2018) 182, [arXiv:1711.08223[hep-th]].

[39] K.B. Alkalaev, M.A. Grigoriev, Continuous spin fields of mixed-symmetry type, JHEP 1803 (2018) 030, [arXiv:1712.02317[hep-th]]; K.B. Alkalaev, A. Chekmenev, M.A. Grigoriev, Unified formulation for helicity and continuous spin fermionic fields, JHEP 11 (2018) 050, [arXiv:1808.09385 [hep-th]].

[40] I.L. Buchbinder, S. Fedoruk, A.P. Isaev, Twistorial and space-time descriptions of massless infinite spin (super)particles and fields, Nucl. Phys. B945 (2019) 114660, [arXiv:1903.07947 [hep-th]].

[41] E.P. Wigner, Relativistische Wellengleichungen, Z. Physik 124 (1947) 665; V. Bargmann, E.P. Wigner, Group theoretical discussion of relativistic wave equations, Proc. Nat. Acad. Sci. US 34 (1948) 211.

[42] C. Burdik, V.K. Pandey, A. Reshetnyak, BRST-BFV and BRST-BV Descriptions for Bosonic Fields with Continuous Spin on $R^{1,d-1}$, Int. J. Mod. Phys. A35 (2020) 26, 2050154, [arXiv:1906.02585[hep-th]].

[43] L.D. Faddeev, V.N. Popov, Feynman Diagrams for the Yang-Mills Field, Phys. Lett. B. 25, 29 (1967).

[44] R.R. Metsaev, BRST invariant effective action of shadow fields, conformal fields, and AdS/CFT, Theor.Math.Phys. 181 (2014) 1548, [arXiv:1407.2601 [hep-th]].

[45] W. Siegel, Introduction to String Field Theory, Adv.Ser.Math.Phys. 8 (1988) 1, [arXiv:hep-th/0107094].

[46] I.L Buchbinder, A. Fotopoulos, A.C. Petkou, M. Tsulaia, Constructing the cubic interaction vertex of higher spin gauge fields, Phys. Rev. D 74 (2006) 105018, [arXiv:hep-th/0609082].

[47] P. Dempster, M.Tsulaia, On the Structure of Quartic Vertices for Massless Higher Spin Fields on Minkowski Background, Phys. Rev. D 86 (2012) 025007, [arXiv:1203.5597[hep-th]].

[48] C. Fronsdal, Massless Fields with Integer Spin Phys. Rev. D18 (1978) 3624.

[49] D. Francia, A. Sagnotti, On the geometry of higher spin gauge fields, Class. Quant. Grav. 20 (2003) S473.

[50] P.M. Lavrov, P.Yu. Moshin, A.A. Reshetnyak, Superfield formulation of the Lagrangian BRST quantization method, Mod Phys. Lett. A. 10 (1995) 2687; JETP Lett. 62 (1995) 780, [arxiv: hep-th/9507104].

[51] D.M. Gitman, P.Yu. Moshin, A.A. Reshetnyak, Local superfield Lagrangian BRST quantization, J. Math. Phys. 46 (2005) 072302, [arXiv:hep-th/0507160]; An Embedding of the BV quantization into an N=1 local superfield formalism, Phys. Lett. B 621 (2005) 295, [arXiv:hep-th/0507049];
A.A. Reshetnyak, The Effective action for superfield Lagrangian quantization in reducible hypergauges, Russ.Phys.J. 47 (2004) 1026, [arXiv:hep-th/0512327].