

# **Booklet of abstracts ISQS25**

## Plenary Talks

# **Quantum Symmetries: from Clifford Numbers to M-Theory and Leech Lattices**

Sultan Catto

CUNY Graduate School and the Rockefeller University, New York

We present a construction scheme for Clifford numbers of arbitrary dimension and illustrate building of color algebras, dynamical supersymmetry and superstrings based on exceptional structures and their embedding to M-theory and Leech lattice.

# Integrable many-body models in action-angle duality from reductions of Heisenberg doubles

László Fehér

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I review recent results on the construction of integrable many-body models by Hamiltonian reduction that generalize the derivation of the Calogero and Sutherland systems due to Kazhdan, Kostant and Sternberg. Symplectic reductions are applied to the Heisenberg double of the standard Poisson  $SU(N)$  group equipped with two Abelian Poisson algebras. These Abelian algebras are generated by invariants of the underlying symmetry group, which descend to action and position variables of two Liouville integrable many-body models of Ruijsenaars–Schneider–van Diejen type in action-angle duality. The global structure of the reduced phase spaces and qualitative features of the dynamical systems will be also described. The talk is based on ongoing joint work with Ian Marshall (see arXiv:1702.06514) and earlier papers.

# R-matrix, star-triangle relations and Yangians for conformal algebras

Alexei Isaev

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In the case of the conformal algebra of  $n$ -dimensional Euclidean space we construct the R-operator for the scalar (spin part is equal to zero) representations and prove that the star-triangle relation is a well known star-triangle relation for propagators of scalar fields. In the special case of the conformal algebra of the 4-dimensional Euclidean space, the R-operator is obtained for more general class of infinite-dimensional (differential) representations with nontrivial spin parts. As a result, for the case of the 4-dimensional Euclidean space, we generalize the scalar star-triangle relation to the most general star-triangle relation for the propagators of particles with arbitrary spins. Special representations of Yangians  $Y(\mathfrak{so}(n))$  and  $Y(\mathfrak{osp}(n|m))$  are discussed.

# Classical and quantum superfield invariants in $6D$ , $\mathcal{N} = (1, 1)$ SYM theory

Evgeny Ivanov

BLTP JINR (Dubna)

We determine the structure of candidate counterterms in  $6D$ ,  $\mathcal{N} = (1, 1)$  SYM theory in the  $\mathcal{N} = (1, 0)$  harmonic superspace approach, combining both the requirement of hidden  $\mathcal{N} = (0, 1)$  supersymmetry and the techniques of direct quantum calculations of harmonic  $\mathcal{N} = (1, 0)$  supergraphs. The talk is essentially based on recent joint papers with G. Bossard, A. Smilga, I. Buchbinder, B. Merzlikin and K. Stepanyants.

# Integrals of motion from quantum toroidal algebras

Michio Jimbo

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We revisit the integrals of motion for deformed Virasoro algebras introduced earlier by Feigin et al. We explain their connection to quantum toroidal algebras, and establish the Bethe ansatz equations for them. Our method is based on representation theory of Borel subalgebras.

# Quantum Q systems, graded characters and quantum current algebras

Rinat Kedem

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I will explain a connection between the graded characters of the Feigin-Loktev fusion product, which can be thought of as a partition function of the conformal spectrum of the generalized Heisenberg spin chain, and a quantum cluster algebra called the quantum Q-system. I will then show that the algebra satisfied by the solutions of this system is part of the quantum affine algebra of  $sl_2$ , and that it is part of a more general structure which has a further deformation into the realm of Macdonald operators.



# Elliptic Painlevé equation and elliptic hypergeometric integrals

Masatoshi Noumi

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I will give an overview of the formulation of tau-functions for the elliptic Painlevé equation (with affine Weyl group symmetry of type  $E_8$ ), and explain how elliptic hypergeometric integrals arise as special solutions in this framework.

# Second quantisation of Dunkl operators

Evgeny Sklyanin

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The 1D system of  $N$  nonrelativistic bosons interacting via delta-function potential is arguably the simplest and very well studied example of a quantum integrable system. Its integrability can be established by a variety of methods, including the coordinate Bethe ansatz and Dunkl operators. In the second quantisation, the corresponding quantum field theory in the Fock space, that is the orthogonal sum of the  $N$ -particle spaces, represents a quantisation of the Nonlinear Schroedinger equation. We study the second quantised version of the Dunkl operators and obtain some new representations for the integrals of motion and creation/annihilation operators both in the quantum case and in the classical limits.

# Soliton classical solutions in the modified Yang-Mills theory

Andrei Slavnov

Steklov Mathematical Institute, Moscow, Russia

The new formulation of the quantization procedure, producing the same formal perturbation theory as the standard one is proposed. It is shown that this formulation produces the soliton solutions of the classical equations.

# Scalar products of Bethe vectors in the models with $gl(m|n)$ symmetry

Nikita Slavnov

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We consider integrable models with  $gl(m|n)$  symmetry solvable by the nested algebraic Bethe ansatz. Using a coproduct properties of Bethe vectors we find an explicit representation for their scalar product as a sum over partitions of Bethe parameters. This representation allows us to prove a generalization of Gaudin formula for the norm of an on-shell Bethe vector.

# HK and HKT geometries with supersymmetric glasses

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We give a review of geometrical properties of hyper-Kaehler manifolds and so-called HKT (hyper-Kaehler with torsion) manifolds. Then we show how the metric of a generic HK or HKT manifold can be reconstructed from certain data, using the methods of supersymmetric quantum mechanics. This procedure is, however, essentially more complicated than for Kaehler manifolds [where the metric is derived from the Kaehler prepotential by a simple differentiation,  $g_{j\bar{k}} = \partial_j \partial_{\bar{k}} \mathcal{K}(z^m, \bar{z}^{\bar{m}})$ ]. For HK and HKT manifolds one should use the technique of harmonic superspace. For example, the geometry of a generic HK manifold is determined by a function  $\mathcal{L}^{+4}$  of harmonic charge +4, which depends on harmonic superfields of unit charge  $q^+$  (which are related to the coordinates) and the harmonics. The metric is then obtained as a result of the solution of rather complicated differential equations. In certain cases, these equations can be solved explicitly, and analytic expressions for the metric can be found.

# Nonassociative deformations of quantum mechanics

Richard Szabo

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I will explain some of the progress that has been achieved over the years in understanding certain noncommutative and nonassociative deformations of quantum theories, and how they may teach us something about the short-scale structure of spacetime, and ultimately quantum gravity. I will emphasise how these theoretical consequences can connect to real-world measurable quantities, and analyse in detail a simple deformation of quantum mechanics which may be realised in a table-top experiment. I will explain how these quantum systems are related to the nonassociative algebras of observables proposed in the beginnings of quantum mechanics and quantum field theory by Jordan, von Neumann, Wigner and others to study the mathematical and conceptual foundations of quantum theory.

# Symmetry of the $XXZ$ model at roots of unity revisited

Vitaly Tarasov

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*and*

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I will discuss the symmetry of the  $XXZ$  model at roots of unity with quasiperiodic boundary conditions given by a certain covering of the semigroup of matrix-valued polynomials. For a special choice of boundary conditions, this symmetry induces the  $\mathfrak{sl}_2$  loop algebra symmetry of the  $XXZ$  model at roots of unity discovered by T. Deguchi, K. Fabricius and B. McCoy in 1999, and generalized later by C. Korff.

# Eigenfunction Asymptotics for Quantum Completely Integrable Systems

John A. Toth

McGill University  
Montreal, Canada

I will discuss the semiclassical asymptotics of joint eigenfunctions of quantum completely integrable systems, including results on  $L^p$ -concentration as well as restriction bounds.



# Current Interactions from Nonlinear Higher-Spin Equations and Holography

Mikhail Vasiliev

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Recent progress in understanding the concept of locality in higher-spin theory will be discussed in the context of derivation of current interactions from the nonlinear higher-spin field equations. Related issues of the holographic correspondence will also be considered.

## Parallel Talks

# Cofree objects in the centralizer and the center categories

Adnan H. Abdulwahid

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We study cocompleteness, co-wellpoweredness and generators in the centralizer category of an object or morphism in a monoidal category, and the center or the weak center of a monoidal category. We explicitly give some answers for when colimits, cocompleteness, co-wellpoweredness and generators in these monoidal categories can be inherited from their base monoidal categories. Most importantly, we investigate cofree objects of comonoids in these monoidal categories.

# **Analytical solutions of the Schrödinger equation for the Manning-Rosen plus Hulthén potential within supersymmetric quantum mechanics**

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Joint work with: M. Naeem, M. V. Qocayeva and V. A. Tarverdiyeva

In this work, the analytical solution of the modified radial Schrödinger equation are studied in great detail for the Manning-Rosen plus Hulthén potential. Within the framework a novel improved scheme to surmount centrifugal term, the energy eigenvalues and corresponding radial wave functions are found for any  $l$  orbital angular momentum case within the context of the Nikiforov-Uvarov and supersymmetric quantum mechanics methods. The energy levels are worked out and the corresponding normalized eigenfunctions are obtained in terms of orthogonal polynomials for arbitrary  $l$  states. In this way, based on these methods, the same expressions are obtained for the energy eigenvalues, and the expression of radial wave functions transformed each other is demonstrated.

# Smooth dense subalgebras and Fourier multipliers on compact quantum groups

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Joint work with: Shahn Majid and Michael Ruzhansky

We define and study dense Frechet subalgebras of compact quantum groups consisting of elements rapidly decreasing with respect to an unbounded sequence of real numbers. Further, this sequence can be viewed as the eigenvalues of a Dirac-like operator and we characterize the boundedness of its commutators in terms of the eigenvalues. Grotendieck's theory of topological tensor products immediately yields a Schwartz kernel theorem for linear operators on compact quantum groups. This allows us to introduce a natural class of pseudo-differential operators on compact quantum groups. As a by-product, we develop elements of the distribution theory and corresponding Fourier analysis. We give applications of our construction to obtain sufficient conditions for  $L^p - L^q$  boundedness of coinvariant linear operators. We provide necessary and sufficient conditions for algebraic differential calculi on Hopf subalgebras of compact quantum groups to agree with the proposed smooth structure.

# Integrable structure of stochastic Laplacian growth

Oleg Alekseev

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The deterministic Laplacian growth possess profound mathematical structure, strongly connected with major integrable hierarchies. We propose its stochastic version to treat *analytically* static and dynamic fluctuations of the growing interface. New connections with the Calogero-Sutherland system, Burgers turbulence, and conformal field theory are revealed.

# Corrections to Newton's law of gravitation – application to hybrid Bloch brane

Carlos Alberto Almeida

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Joint work with: Diego Veras

We present in this work, the calculations of corrections in the Newton's law of gravitation due to Kaluza-Klein gravitons in five-dimensional warped thick braneworld scenarios. We consider here some recently proposed model, namely, the hybrid Bloch brane. This model couples two scalar fields to gravity and is engendered from a domain wall-like defect. Also, two other models the so-called asymmetric hybrid brane and compact brane are considered. Such models are deformations of the  $\phi^4$  and sine-Gordon topological defects, respectively. Therefore we consider the branes engendered by such defects and we also compute the corrections in their cases. In order to attain the mass spectrum and its corresponding eigenfunctions which are the essential quantities for computing the correction to the Newtonian potential, we develop a suitable numerical technique. Moreover, we discuss that the existence of massive modes is necessary for building a braneworld model with a phenomenology involved. The calculation of slight deviations in the gravitational potential may be used as a selection tool for braneworld scenarios matching with future experimental measurements in high energy collisions.

# Modular properties of 6d (DELL) systems

G. Aminov, A. Mironov, A. Morozov

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If super-Yang-Mills theory possesses exact conformal invariance, there is an additional modular invariance under the change of the complex bare charge  $\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2} \longrightarrow -\frac{1}{\tau}$ . The low-energy Seiberg-Witten prepotential  $\mathcal{F}(a)$ , however, is not explicitly invariant, because the flat moduli also change  $a \longrightarrow a_D = \partial\mathcal{F}/\partial a$ . In result the prepotential is not a modular form, depending also on anomalous Eisenstein series  $E_2$ . This dependence is usually described by the universal MNW modular-anomaly equation. We demonstrate, that in 6d theory with *two* independent modular parameters  $\tau$  and  $\hat{\tau}$  modular anomaly changes, because the modular transform of  $\tau$  is accompanied by an ( $N$ -dependent!) shift of  $\hat{\tau}$  and vice versa. This is a new peculiarity of double-elliptic systems, which deserves further investigation.



# Noncommutative Quantum Mechanics based on Representations of Exotic Galilei Group

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Using elements of symmetry, we constructed the Noncommutative Schrödinger Equation from a representation of Exotic Galilei Group. As a consequence, we derive the Ehrenfest theorem using noncommutative coordinates. We also have showed others features of quantum mechanics in such a manifold. As an important result, we find out that a linear potential in the noncommutative Schrödinger equation is completely analogous to the ordinary case. We also worked with harmonic and anharmonic oscillators, giving corrections in the energy for each one.

# What is a Symmetry Witness?

Paolo Aniello

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We will introduce a notion of ‘symmetry witness’, related to the symmetry transformations of quantum mechanics. A symmetry witness is a set of selfadjoint trace class operators that allows one to ascertain whether a linear map is a symmetry transformation. More precisely, such a set is invariant with respect to an injective linear operator in the Banach space of selfadjoint trace class operators if and only if this operator is a symmetry transformation. By a linear version of Wigner’s classical theorem, the set of pure states — the rank-one projections — is a symmetry witness. We stress that linearity entails that the usual assumption of preservation of the transition probabilities becomes superfluous. This result extends to every set of projections with a fixed (finite) rank, with some suitable constraint on this rank. One then obtains a classification of the sets of projections of a fixed rank that are symmetry witnesses. These symmetry witnesses are projectable. Namely, thinking in terms of quantum states, the sets of ‘uniform’ density operators of a (suitable) fixed rank are symmetry witnesses as well.

# A $q, t$ -Integrable System on a Genus Two Surface

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Joint work with: Sh. Shakirov (Harvard)

In my talk I will consider a quantum integrable system with two generic complex parameters  $q, t$  whose classical phase space is the moduli space of flat  $SL(2, \mathbb{C})$  connections on a genus two surface. This system and its eigenfunctions provide genus two generalizations of the trigonometric Ruijsenaars-Schneider model and  $A_1$  Macdonald polynomials, respectively. I will show that the Mapping Class Group of a genus two surface acts by outer automorphisms of the algebra of operators of this system. Therefore this algebra can be viewed as a genus two generalization of  $A_1$  spherical DAHA. (Based on arXiv:1704.02947)

# Emergent five-sphere from the BMN Matrix model

Yuhma Asano

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Explicit realisation of transverse M5-branes in matrix models has been a difficult problem. However, in the BMN matrix model, M2- and M5-brane states are known to be identified with vacua in the matrix model. In this talk, based on this identification, we consider trivial vacua, which are considered to describe a single M5-brane in M-theory on the maximally supersymmetric plane-wave background. I will explain how the eigenvalue distribution of a BPS operator, which is suitable for describing the low-energy region, forms the five-sphere wrapped by the M5-brane, by using the results of exact computation of the BPS observable.

# An extension of Drosel-Schwable Forest-fire model on Bethe lattice

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Joint work with: Mariusz Białecki

To construct this model we will use the concept of the Forest-fire model (Drosel-Schwable) and Random Domino Automaton (RDA). Random Domino Automaton (RDA) is a completely discrete stochastic dynamical system which model slow accumulation of energy and its abrupt releases. Originally it was defined for 1D line geometry. Here I will present a generalization of RDA for Bethe lattice in the context of the forest-fire model and point out complications related to this more complex geometry. I will show how to use properties of Bethe lattice in order to introduce suitable set of variables describing stationary state of the system. Finally, I will show how to obtain a closed set of equations for this nontrivial set of variables and possible solution.

# Symmetries within fractional calculus

Dumitru Baleanu

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Fractional order differential equations, have assumed an important role in modelling the anomalous dynamics of numerous processes related to complex systems in the most diverse areas of science and engineering. In my presentation I will discuss the concept of symmetries within the fractional calculus.

# Generalized Weyl algebras and diskew polynomial rings

V.V. Bavula

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The aim of the paper is to extend the class of generalized Weyl algebras to a larger class of algebras (they are also called *generalized Weyl algebras*) that are determined by two algebra endomorphisms rather than one as in the case of classical GWAs. A new class of rings, the *diskew polynomial rings*, is introduced that is closely related to GWAs. The, so-called, ambiskew polynomial rings are a small subclass of the class of diskew polynomial rings. Semisimplicity criteria are given for generalized Weyl algebras and diskew polynomial rings. Various properties and examples of these algebras are considered. Many classical algebras and quantum groups are examples of GWAs and diskew polynomial rings.

# Quantum nonlocality, Aristotle, and the nature of time

Charlie Beil

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I will describe a possible new framework for quantum theory that is based on incorporating Aristotle's notion of time into relativity. During the fourth century BC, Aristotle proposed that time passes if and only if something changes. This operational notion of time may be obtained by applying the identity of indiscernibles, which is a fundamental principle of quantum statistics, to time. By appropriately modifying this notion into a Hilbert space setting, I will describe a new geometry of relativistic spacetime where events may be positive dimensional. This geometry naturally exhibits quantum nonlocality, and is realized algebraically as a nonnoetherian scheme. I will outline consequences of this framework for ultraviolet divergences and Wick rotations in quantum field theory.



## About $q$ -Onsager algebra and others coideal subalgebras

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We will discuss coideal subalgebras of the quantum group  $U_q(\widehat{sl_2})$  focussing on the case of the  $q$ -Onsager algebra. Mathematical aspects and physical applications will be given.

# $q$ -deformed Painlevé $\tau$ function and $q$ -deformed conformal blocks

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Joint work with: A. Shchekkin

We propose  $q$ -deformation of the Gamayun-Iorgov-Lisovyy formula for Painlevé  $\tau$  function. Namely we propose the formula for  $\tau$  function for  $q$ -difference Painlevé equation corresponding to  $A_7^{(1)'}$  surface (and  $A_1^{(1)}$  symmetry) in Sakai classification. In this formula  $\tau$  function equals the series of  $q$ -Virasoro Whittaker conformal blocks (equivalently Nekrasov partition functions for pure  $SU(2)$  5d theory).

# On the supersymmetric Fokas-Gel'fand immersion formula for soliton supermanifolds

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short Abstract: In this presentation, we investigate a geometric characterization for the supersymmetric versions of the Fokas-Gel'fand formula for the immersion of soliton supermanifolds with two bosonic and two fermionic independent variables into Lie superalgebras. The approach used considers three types of linear spectral problems: the first using covariant derivatives, the second using bosonic variable derivatives and the third using fermionic variable derivatives. We demonstrate that the second and third types of linear spectral problems can be obtained from the first type via a specific relation. This allows us to investigate, through the first and second fundamental forms, the geometry of the supermanifolds immersed in Lie superalgebras. Whenever possible, the mean and Gaussian curvatures of the supermanifolds are calculated. These theoretical considerations are applied to the supersymmetric sine-Gordon equation.

# On extensions of 1D Drossel-Schwabl forest-fire model

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Joint work with: Arpan Bagchi

We reformulate widely known 1D forest-fire model by Drossel and Schwabl in terms of energy accumulation and release in order to model basic statistical properties of earthquakes. We introduce parameters related to probability of release of energy depending on its amount as well as parameters related to accumulation of energy depending on its amount in a neighborhood. We demonstrate that for some class of extensions the distribution of accumulated energy for stationary state of the system is related to Motzkin numbers.

We discuss also a construction and properties of similar cellular automaton related to Catalan numbers.

# Degenerative discrete-diffractive Bethe ansatz

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An extended version of Bethe ansatz elaborated especially for treatment of non-integrable systems is suggested. The general isotropic  $S = 1$  ferromagnetic chain is studied as an example.

# Noncommutative Riemann surfaces

Pierre Bieliavsky

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Abstract: I will present a construction of noncommutative differentiable (in the sense of Alain Connes) orientable compact surfaces in every higher genus  $g > 0$ . When the genus  $g$  is one, the construction reduces to the usual noncommutative differentiable torus defined by Marc A. Rieffel in the nineties.

# Integrable Hénon–Heiles systems on curved spaces: The Sawada–Kotera case

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Amongst the three (Liouville) integrable Hénon–Heiles systems on the two-dimensional Euclidean plane, we consider the Sawada–Kotera Hamiltonian given by

$$\mathcal{H} = \frac{1}{2}(p_1^2 + p_2^2) + \delta(q_1^2 + q_2^2) + \gamma \left( q_1^2 q_2 + \frac{1}{3} q_2^3 \right),$$

where  $\delta$  and  $\gamma$  are two arbitrary real parameters. This system corresponds to the superposition of an isotropic oscillator (whenever  $\delta > 0$ ) and a cubic potential. It is well-known that the corresponding integral of motion  $\mathcal{I}$  is quadratic in the momenta and  $\mathcal{H}$  becomes separated in rotated Cartesian coordinates:  $u = q_1 + q_2$  and  $v = q_1 - q_2$ .

In this contribution, we present the constant curvature analogue of  $\mathcal{H}$ , denoted  $\mathcal{H}_\kappa$ , on the two-dimensional sphere and the hyperbolic plane, where the real parameter  $\kappa$  is just the (Gaussian) curvature of the underlying Riemannian space. In our approach, the curvature  $\kappa$  can be regarded as a deformation/contraction parameter, in such a manner that the Euclidean Hénon–Heiles system  $\mathcal{H}$  is recovered when the flat limit  $\kappa \rightarrow 0$  is performed.

In particular, we explicitly obtain the curved constant of motion  $\mathcal{I}_\kappa$  (which remains as a quadratic one in the momenta) and, furthermore, we solve the Hamilton–Jacobi separability of the curved system on an appropriate curved counterpart of the rotated Cartesian coordinates  $(u, v)$ .

We stress that the separability of  $\mathcal{H}_\kappa$  leads to a new series of integrable (curved) homogeneous potentials, similarly to what happens with the Ramani–Dorizzi–Grammaticos homogeneous potentials which are related to the Hénon–Heiles KdV Hamiltonian.

## References

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# Higher order superintegrable systems: a new Painlevé conjecture

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The *Painlevé conjecture* has played an important role in the theory of infinite dimensional integrable systems (soliton theory) where it has been highly successful in identifying equations solvable by the inverse scattering transform. Here we present and justify a different Painlevé conjecture in the theory of finite-dimensional quantum superintegrable systems. We present a review of some recent results on "exotic quantum potentials" in two-dimensional Euclidean space that allow the separation of variables in the Schroedinger equation and allow an additional integral of motion of order  $N > 2$ . We call them exotic because they do not satisfy any linear PDE. Instead, they satisfy nonlinear ODEs. For  $N = 3, 4$  and  $5$  it turns out that these nonlinear ODEs always have the Painlevé property. We conjecture that this true for all values of  $N$ .



# BPS monopoles with non-minimal gauge interactions

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We study magnetic monopoles in theories with non-minimal scalar-gauge interactions with both adjoint and fundamental fields. We find a large class of models which admit BPS magnetic monopoles and generalize the ordinary 't Hooft-Polyakov theory. The impact of extra terms on basic properties of the monopole is investigated.

# Pedal coordinates, Dark Kepler and other force problems

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We will make the case that *pedal coordinates* (instead of polar or Cartesian coordinates) are more natural settings in which to study force problems of classical mechanics in the plane. We will show that the trajectory of a test particle under the influence of central and Lorentz-like forces can be translated into pedal coordinates at once without the need of solving any differential equation. This will allow us to generalize Newton theorem of revolving orbits to include nonlocal transforms of curves. Finally, we apply developed methods to solve the “dark Kepler problem”, i.e. central force problem where in addition to the central body, gravitational influences of dark matter and dark energy are assumed.

# Quantum dynamics of coherent states under strong magnetic field

Gregory Boil

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We consider a charged particle subject to a strong, purely magnetic external field. It is well known that for short times, the quantum dynamics closely follows the classical one. Using a recent completely integrable normal form of Raymond and Vu Ngoc, we prove that for very long times, the quantum evolution of a certain class of coherent states follows the average dynamics of the guiding center motion.

# Twisted bialgebroids as a model of quantum phase space

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Joint work with: A. Pachol

Drinfeld twist techniques are particularly useful in (deformation) quantization of Lie algebras as well as underlying module algebras (quantum spaces). Crossed (smash) product construction combines these two into a new algebra which, in fact, does not depend on the twist. However, we can turn it into bialgebroid in the twist dependent way. We argue that within this framework one obtains quantized phase space.

# Chern–Simons theory, topological strings, and relativistic integrable systems

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United Kingdom

Joint work with: G. Borot, B. Eynard, M. Mariño, and further work in progress

I will report on recent progress on the Gopakumar–Ooguri–Vafa correspondence, relating Chern–Simons theory and quantum invariants of 3-manifolds to topological strings and curve-counting theories (Gromov–Witten/Donaldson–Thomas) of local Calabi–Yau threefolds, in the context of Seifert-fibred 3-manifolds. I will describe mirror symmetry constructions for the correspondence in the broadest context where the standard form of the duality is expected to hold (spherical space forms), discuss the link with relativistic integrable systems and the Eynard–Orantin topological recursion, and present a rigorous proof of the B-model version of the correspondence via matrix model techniques. Implications for refined invariants, orbifold GW theory, and an allied class of Frobenius manifolds and 2D-Toda reductions will be also discussed time permitting.

# A representation basis for the quantum integrable spin chain associated with the $\mathfrak{su}(3)$ algebra

Junpeng Cao

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Joint work with: Kun Hao, Guang-Liang Li, Wen-Li Yang, Kangjie Shi and Yupeng Wang

An orthogonal basis of the Hilbert space for the quantum spin chain associated with the  $\mathfrak{su}(3)$  algebra is introduced. Such kind of basis could be treated as a nested generalization of separation of variables basis for high-rank quantum integrable models. It is found that all the monodromy-matrix elements acting on a basis vector take simple forms. With the help of the basis, we construct eigenstates of the  $\mathfrak{su}(3)$  inhomogeneous spin torus (the trigonometric  $\mathfrak{su}(3)$  spin chain with antiperiodic boundary condition) from its spectrum obtained via the off-diagonal Bethe Ansatz. Based on small sites check, it is conjectured that the homogeneous limit of the eigenstates exists, which gives rise to the corresponding eigenstates of the homogenous model.

# Algebraic structures in exceptional field theory

Martin Cederwall

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Chalmers Univ. of Technology  
Gothenburg, Sweden

Exceptional field theory (EFT) gives a geometric underpinning of the U-duality symmetries of M-theory. In this talk I give an overview of the surprisingly rich algebraic structures which naturally appear in the context of EFT. This includes Borcherds superalgebras, Cartan type superalgebras (tensor hierarchy algebras) and  $L_\infty$  algebras.

# Multiplicative quiver varieties and generalized Ruijsenaars–Schneider systems

Oleg Chalykh

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Joint work with: Maxime Fairon

We will explain how the Ruijsenaars-Schneider model and its generalizations can be derived from the space of representations of a framed cyclic quiver, by using the method of quasi-Hamiltonian reduction. The main tool is the notion of a double (quasi-)Poisson bracket and noncommutative multiplicative moment map, due to M. Van den Bergh, with the integrability of the constructed systems manifesting itself already at the noncommutative algebra level. This is based on [arXiv:1704.05814](https://arxiv.org/abs/1704.05814) [math.QA].



# Ordering of energy levels for one-dimensional interacting Fermi gases with $SU(n)$ symmetry

Shu Chen

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Based on the exact solution of one-dimensional Fermi gas systems with  $SU(n)$  symmetry, we demonstrate that it is possible to sort the ordering the lowest energy eigenvalues of states with all allowed permutation symmetries, which goes beyond the generalized Lieb-Mattis theorem.

# Generalizations of the semi- and fully discrete Lotka-Volterra lattice

Xiaomin Chen<sup>1</sup>, Xiangke Chang<sup>2</sup>, Xingbiao Hu<sup>2</sup>

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It is well known that the integrable semi-discrete and fully discrete Lotka-Volterra equations possess special Lax pairs involving symmetric orthogonal polynomials. We construct a nonisospectral semi-discrete Lotka-Volterra equation and propose a more general solution of the fully discrete Lotka-Volterra equation. The key point is introducing a more general evolution relation for the moments of symmetric orthogonal polynomials, which involves a “convolution term”. Furthermore, we generate corresponding exact (“molecule”) solutions, expressed in terms of Hankel-type determinants. Our approach makes use of Hirota bilinear method and determinant techniques.

# Vibron transport in macromolecular chains with squeezed phonons

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Joint work with: Dalibor Čevizović, Slobodanka Galović  
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We investigate physical properties of a single vibronic intramolecular excitation propagating through a macromolecule, whose vibrational state can be described as a squeezed vacuum state. For a theoretical description of such a process, the partial dressing method of the vibronic excitation due to its interaction with phonons is used. We study the influence of the model parameters and strength of squeezing on the vibron dressing. It is shown that for certain values of the model parameters a polaron crossover can occur, in which there is a sharp change in the migration nature of a vibron from practically free to a heavy quasiparticle dressed by a phonon cloud.

# On a Microscopic Representation of Space-Time VII

## – Lines and Spin

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We recall some basic aspects of line and line Complex representations, of symplectic symmetry emerging in bilinear point transformations as well as of Lie transfer of lines to spheres. Here, we identify  $SU(2)$  spin in terms of (classical) projective geometry and obtain spinorial representations from lines, i.e. we find a natural nonlocal geometrical description associated to spin. We discuss the construction of a Lagrangean in terms of line/Complex invariants, and show that the action of the Dirac algebra  $\{\gamma^\mu\}$  acting on  $2 \times 4$  (real) line representations maps (real) line coordinates to (real) line coordinates, thus (up to reflections) it maps and/or interchanges the (line) coordinates,  $p_{\alpha\beta} \longrightarrow \pm p_{\gamma\delta}$ , of the edges of the fundamental tetrahedron which allows to associate  $SU(4)$  and its various real forms covering  $SO(n,m)$ ,  $n + m = 6$ .

# Cohomological methods in immersion formulas

J. de Lucas

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A geometric approach to immersion formulas for soliton surfaces is provided via a generalisation of the de Rham cohomology and differential to a space of Lie algebra-valued differential forms parametrized by a spectral parameter. This leads to the introduction of new Poincare-type lemmas for these cohomologies, which appropriately describe integrability conditions and deformations of Lax pairs. In this language, properties of soliton surfaces become very simple and generalizations of 2D-models to soliton submanifolds appear straightforwardly. Theoretical results are illustrated by physical examples.

# Almost homogeneous Schrödinger operators

Jan Dereziński

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First I will describe a certain natural holomorphic family of closed operators with interesting spectral properties. These operators can be fully analyzed using just trigonometric functions.

Then I will discuss 1-dimensional Schrödinger operators with a  $1/x^2$  potential with general boundary conditions, which I studied recently with S.Richard. Even though their description involves Bessel and Gamma functions, they turn out to be equivalent to the previous family.

Some operators that I will describe are homogeneous — they get multiplied by a constant after a change of the scale. In general, their homogeneity is weakly broken — scaling induces a simple but nontrivial flow in the parameter space. One can say (with some exaggeration) that they can be viewed as "toy models of the renormalization group".

# Evolution of Landau levels in graphene-based topological insulators in the presence of wedge disclinations

de Souza Oliveira

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We consider the modification of the electronic properties of the graphene-based topological insulation in the presence of a wedge-shaped disclination (*wedge disclination*) and in the presence of a magnetic field admitting the Kane-Mele model with spin-orbit coupling, in addition to study the effects of the presence of a mass term as studied in the Haldane model. We use the Dirac-Weyl equation properly defined for this system, thus obtaining an exact solution for the self-values of the Landau levels. We thus discuss the influence of topological defects on the evolution of Landau levels.

# On a geometric solution of Zamolodchikov's tetrahedron equation: from noncommutative to quantum

Adam Doliwa

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Starting from four dimensional consistency of the noncommutative discrete KP system in its Desargues maps (homogeneous) form we present the corresponding solution to Zamolodchikov's tetrahedron equation. Then, by applying ultralocal reduction, we give its quantum version.



# Defect in the Joint Spectrum of Hydrogen due to Monodromy

Holger R. Dullin

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University of Sydney  
Sydney NSW 2006  
Australia

Joint work with: Holger Waalkens

We show that the joint spectrum of the Hamilton operator of the Kepler problem, the  $z$ -components of the angular momentum, and the  $z$ -components of the Laplace-Runge-Lenz vector obtained from separation in prolate spheroidal coordinates has quantum monodromy for energies sufficiently close to the ionization threshold. The principal quantum number  $n$  and the magnetic quantum number  $m$  correspond to the Bohr-Sommerfeld quantization of globally defined classical actions, but a third quantum number cannot be globally defined because the third action is globally multi valued.

# Geometric deautonomization of a QRT mapping and elliptic difference Painlevé equations

Anton Dzhamay

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Joint work with: Adrian Stefan Carstea and Tomoyuki Takenawa

This talk is the second of two talks describing the joint project with Stefan Carstea and Tomoyuki Takenawa on geometric deautonomization. In this talk we illustrate the general deautonomization approach by describing in detail a deautonomization of a simple QRT mapping using a choice of an elliptic fiber. First we show how to explicitly construct a discrete non-autonomous mapping whose action on the Picard lattice of the surface is the same as the original QRT action. This mapping is, however, not an elliptic difference equation since the coefficients of the equation do not change translationally in the argument of some elliptic function, only the fourth power of it is. However, by imposing additional constraints on the parameter evolution gives special cases of the mapping such that either the square of the mapping or the mapping itself become elliptic difference equations. The resulting equations are new and they also can be written in a simple explicit form. The symmetry groups of these equations are special in the sense that they are described by Dynkin diagrams that do not appear explicitly in the Sakai's classification scheme (w.r.t. the symmetry type of the equation), which only concerns the generic situation. Our examples clarify the role of such exotic symmetry groups in the theory of discrete Painlevé equations.

# Fractional Hamiltonian Monodromy

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Joint work with: N. Martynchuk

Standard Hamiltonian monodromy was introduced by J. J. Duistermaat in 1980 as an obstruction to the existence of global action coordinates in integrable Hamiltonian systems. It is moreover related to the structure of the joint spectrum of quantum systems. Fractional Hamiltonian monodromy, introduced by N. N. Nekhoroshev, D. A. Sadovskii, and B. I. Zhilinskiĭ in 2004, generalizes standard monodromy by considering not only torus bundles but also more general fibrations with singular fibers.

In this talk we present results concerning fractional monodromy that were recently obtained in collaboration with N. Martynchuk. In particular, we show how in integrable Hamiltonian systems with a Hamiltonian circle action both standard and fractional monodromy can be solely determined through a careful study of the fixed points of the circle action and their weights.

# Integrability in AGT duality

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In modern theoretical physics, a prominent role is covered by the concepts of integrability and conformal invariance. On one side, integrable systems appear in many settings and present a very particular behaviour due to the large extent of underlying symmetries. On the other side, field theories based on conformal invariance are an ubiquitous topic, emerging in the physics of critical phenomena and in string theory. Due to their large amount of symmetries, they are ideal subjects where to study its consequences in light of integrability. The two-dimensional case is an extreme example of this idea, since the conformal algebra is infinite-dimensional. Recently, 2d CFTs earned a renewed interest due to the so-called AGT duality, which relates 2d Liouville CFTs on suitable Riemann surfaces and  $\Omega$ -deformed Seiberg-Witten gauge theories. We present some recent results in the context of AGT duality. We studied the chiral ring structure for the  $\mathcal{N} = 2$  pure and  $\mathcal{N} = 2^*$  Seiberg-Witten theories and how it gets deformed once that the  $\Omega$ -background is switched on, interpreting the resulting structure on the CFT side in view of AGT duality. We also explored the  $SU(2)$   $N_f = 4$   $\Omega$ -deformed Seiberg-Witten theory for fixed values of the matter masses and one of the deformation parameters and required a simple polar structure for the Nekrasov partition function. In this way, we were able to determine it in closed form, giving non-trivial predictions for the conformal block on the 4-punctured sphere.

# **N=4 supersymmetric U(2)-spin Calogero-Moser model as a gauged matrix mechanics**

Sergey Fedoruk

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Joint work with: Evgeny Ivanov

New models of the  $SU(2|1)$  supersymmetric mechanics based on gauging the systems with dynamical and semi-dynamical supermultiplets are presented.  $N = 4$  extension of  $d = 1$  Calogero-Moser multiparticle system is obtained by gauging the  $U(n)$  isometry of matrix  $SU(2|1)$  harmonic superfield model.

# Poisson Algebras and 3D Superintegrable Hamiltonian Systems

Alan Fordy

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Joint work with: Qing Huang

Using a Poisson bracket representation, in 3D, of the Lie algebra  $\mathfrak{sl}(2)$ , we first use highest weight representations to embed this in larger Lie algebras. These are then interpreted as symmetry and conformal symmetry algebras of a Hamiltonian, related to a Casimir function. We then classify the potentials which can be added, whilst remaining integrable and further consider the specialisation to super-integrable cases.

# Integrable lattice spin models from supersymmetric gauge theories

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Recently, there has been observed several connections of integrable models to supersymmetric gauge theories. One of such connections is a correspondence between supersymmetric quiver gauge theories and integrable lattice models such that the integrability emerges as a manifestation of supersymmetric dualities. Particularly, partition functions of supersymmetric quiver gauge theories with four supercharge on different manifolds can be identified with partition functions of two-dimensional exactly solvable statistical models. This relationship has led to the construction of new exactly solvable models of statistical mechanics, namely the Yang-Baxter equation was solved in terms of new special functions. The talk will highlight a few solutions of the Yang-Baxter equation constructed via this relationship.

# Local Current Interactions from Nonlinear Higher-Spin Equations in AdS4

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The form of higher-spin current interactions in the sector of one-forms is derived from the nonlinear higher-spin equations in  $AdS_4$ . Quadratic corrections to higher-spin equations are shown to be independent of the phase of the parameter  $\eta = \exp i\varphi$  in the full nonlinear higher-spin equations. Quadratic current deformation resulting from the nonlinear higher-spin equations is represented in the canonical form with the minimal number of space-time derivatives.



# Umbral Calculus

K. Górska

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Symbolic methods of umbral nature play an important and increasing role in the theory of special functions. I will discuss an application of these methods to polynomials and functions for which we give a number of new closed form expressions. I will present furthermore the different possibilities offered by the method we have developed, with particular emphasis on their link to a new family of special functions and with previous formulations, associated with the theory of quasi monomials.

# Exact solution of quantum field theory toy models

Harald Grosse

University Vienna

Matrix models share all interesting features of quantum field theory: graphical description and Feynman rules, power counting dimension, regularisation and renormalisation, divergence of the perturbation series. We report on models, where much more is possible: We give exact non-perturbative formulae for all renormalised correlation functions. We describe also a map which projects these matrix correlation functions to Schwinger functions of an ordinary quantum field theory. The Schwinger 2-point functions satisfies in two of these models the Osterwalder-Schrader axioms.- These results were obtained in joint work with Raimar Wulkenhaar and partly with Akifumi Sako.

# The parametric bases for the elliptic boundary value problems

A.A. Gusev<sup>1</sup>, S.I. Vinitsky<sup>1,2</sup>, O. Chuluunbaatar<sup>1,3</sup>, G. Chuluunbaatar<sup>1,2</sup>, A. Gózdź<sup>4</sup>

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<sup>4</sup>Institute of Physics, University of M. Curie-Skłodowska, Lublin, Poland

In the talk we consider calculation schemes in the framework of Kantorovich method consisting in a reduction of a 3D elliptic boundary-value problem (EBVP) to a system of second order ordinary differential equations (SSODEs) using the parametric basis functions which are solution of the 2D parametric EBVP. The coefficients of the SSODEs are parametric eigenvalues and potential matrix elements given by integrals of the eigenfunctions multiplied by their first derivatives with respect to the parameter. We calculate the parametric basis functions numerically in a general case using the high-accuracy finite element method, and analytically in some special choices of the 2D parametric EBVP, in particular, for oscillator potentials with variable minima positions and frequencies. We demonstrate an efficiency of the proposed calculation schemes and algorithms for solving the EBVPs describing Helium atom bound states and a quadrupole vibration collective nuclear model.

# Integrable families of hard-core particles with unequal masses in a one-dimensional harmonic trap

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Joint work with: Maxim Olshanii, A.S. Dehkharghani, A.G. Volosniev, Steven Glenn Jackson, N.T. Zinner

We show that the dynamics of particles in a one-dimensional harmonic trap with hard-core interactions can be solvable for certain arrangements of unequal masses. For any number of particles, there exist two families of unequal mass particles that have integrable dynamics, and there are additional exceptional cases for three, four and five particles. The integrable mass families are classified by Coxeter reflection groups and the corresponding solutions are Bethe ansatz-like superpositions of hyperspherical harmonics in the relative hyperangular coordinates that are then restricted to sectors of fixed particle order. We also provide evidence for superintegrability of these Coxeter mass families and conjecture maximal superintegrability.

# Maximally extended $\mathfrak{sl}(2|2)$ , $q$ -deformed $\mathfrak{d}(2, 1; \epsilon)$ and 3D kappa-Poincaré

Reimar Hecht

ETH Zurich

Joint work with: Niklas Beisert and Ben Hoare

We show that the maximal extension  $\mathfrak{sl}(2) \times \mathfrak{psl}(2|2) \times \mathbb{C}^3$  of the  $\mathfrak{sl}(2|2)$  superalgebra can be obtained as a contraction limit of the semi-simple superalgebra  $\mathfrak{d}(2, 1; \epsilon) \oplus \mathfrak{sl}(2)$ . We reproduce earlier results on the corresponding  $q$ -deformed Hopf algebra and its universal R-matrix by means of contraction. We make the curious observation that the above algebra is related to kappa-Poincaré symmetry. When dropping the graded part  $\mathfrak{psl}(2|2)$  we find a novel one-parameter deformation of the 3D kappa-Poincaré algebra. Our construction also provides a concise exact expression for its universal R-matrix.

# A Quantum subgroup depth

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Joint work with: Lars Kadison, Samuel Lopes

The Green ring of the half quantum group  $H = U_n(q)$  is computed in Chen, Van Oystaeyen and Zhen (2014). The tensor product formulas between indecomposables may be used for a generalized subgroup depth computation in the setting of quantum groups to compute depth of the Hopf subalgebra  $H$  in its Drinfeld double  $D(H)$ . In this paper the Hopf subalgebra quotient module  $Q$  is computed and, as  $H$ -modules,  $Q$  and its second tensor power are decomposed into a direct sum of indecomposables. We note that the least power  $n$ , referred to as depth, for which  $Q^{\otimes(n)}$  has the same indecomposable constituents as  $Q^{\otimes(n+1)}$  is  $n = 2$ , since  $Q^{\otimes(2)}$  contains all  $H$ -module indecomposables, which determines the minimum even depth  $d_{ev}(H, D(H))$ .

# Noncommutative spacetimes with cosmological constant

Francisco J. Herranz

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Joint work with: A. Ballesteros, I. Gutierrez-Sagredo

Noncommutative spacetimes are widely believed to model some properties of the quantum structure of spacetime at the Planck regime. The construction of (2+1)-dimensional (anti-)de Sitter noncommutative spacetimes, which are compatible with the Chern-Simons approach to (2+1) gravity, is briefly reviewed. Next, a new (3+1)-dimensional (anti-)de Sitter noncommutative spacetime, coming from a quantum deformation of a Drinfel'd double, is presented and some of its physical properties are outlined. In this approach, the quantum deformation parameter is related to a Planck scale, and the cosmological constant  $\Lambda$  plays the role of a second deformation parameter of a geometric nature, whose limit  $\Lambda \rightarrow 0$  provides the corresponding noncommutative Minkowskian spacetimes.

# On superintegrable monopole systems

Md Fazlul Hoque

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Joint work with: Ian Marquette and Yao-Zhong Zhang

Superintegrable systems with monopole interactions in flat and curved spaces have attracted much attention. For example, models in spaces with a Taub-NUT metric are well-known to admit the Kepler-type symmetries and provide non-trivial generalizations of the usual Kepler problems. In this talk, we first overview new families of superintegrable Kepler, MIC-harmonic oscillator and deformed Kepler systems interacting with Yang-Coulomb monopoles in flat and curved Taub-NUT spaces. We present their higher-order, algebraically independent integrals of motion via direct and constructive approaches which show the superintegrability of the models. The integrals form symmetry algebras of the corresponding Hamiltonians with structure constants involving Casimir operators of certain Lie algebras. Such algebraic approaches provide a deeper understanding to the degeneracies of the energy spectra and the connection between the wave functions and differential equations and geometry. We also present some of the results of our recent works on extended Kepler-Coulomb quantum superintegrable and quasi-exactly solvable monopole systems from exceptional orthogonal polynomials.



# Calculation of the critical indices in strongly correlated electrons system

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We consider the method of calculation the critical indices in system of strong correlated electrons such as t-J model and some modification of the Hubbard model using algebraic Bethe ansatz. Some preliminary results are considered. This method is also appropriate for calculation of critical indices in multi component Bose gases.

# A hierarchy of Schrödinger potentials solvable via a two-term Hermite-function ansatz

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We introduce a hierarchy of potentials for which the solution of the Schrödinger equation is written in terms of the Hermite functions. The hierarchy is derived by a two-term ansatz written as a linear combination with polynomial coefficients of two Hermite functions of non-integer order. The hierarchy of the potentials which all belong to the bi-confluent Heun class starts with the classical harmonic oscillator potential plus the potential of a uniform field and the two conditionally integrable potentials by Stillingner. For these three potentials the coefficient of the second term is equal to zero so that the solution is actually one-term. The fourth and fifth potentials of the list for which the coefficients of the ansatz are constants, have been first indicated by Exton. The hierarchy is presumably infinite. We explicitly list the first eleven potentials. Six of these potentials are derived for the first time. As a representative example, we consider the solution for the ninth potential of the hierarchy which is an infinite well defined on a half-axis. We discuss several examples of generating apparent singular points as a result of differentiating homogeneous linear ordinary differential equations with polynomial coefficients. As particular examples of application of the equations involving apparent singularities, we consider the reduction of the one-dimensional Schrödinger equation to the deformed double-, bi- and tri-confluent Heun equations. We show that this leads to the solution for the inverse square root, the Lambert-W step and the Lambert-W singular potentials via the two-term Hermite function ansatz.

# Quantization of noncompact coverings and its physical applications

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A rigorous algebraic definition of noncommutative coverings is developed. In the case of commutative algebras this definition is equivalent to the classical definition of topological coverings of locally compact spaces. The theory has following nontrivial applications:

- Coverings of the quantum  $SU(2)$  group,
- Coverings of isospectral deformations of Spin manifolds,
- Coverings of noncommutative tori.

The theory supplies the rigorous definition of noncommutative Wilson lines.

# Lie algebra as an application of logarithmic representation of infinitesimal generators

Yoritaka Iwata

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Department of Mathematics, Shibaura Institute of Technology, Japan

A logarithm representation of operators is introduced in Y. Iwata: *Infinitesimal generators of invertible evolution families*, Methods Funct. Anal. Topology **23** 1 (2017) 26, and it is utilized to characterize infinitesimal generators of Lie group. Much attention is paid to discuss a certain kind of reducible unboundedness and nonlinearity of infinitesimal generators.

# Entanglement caused by excitation in 1D spin and electron systems

Dorota Jakubczyk

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We show that entanglement in one-dimensional spin and electron systems, with one excitation, depends only on the system size and has very simple form in both multipartite and bipartite case. We consider the exact solutions given by the Fourier transform known in literature as the basis of wavelets. Regarding the multipartite case, we present very simple expressions for global entanglement and N-concurrence, and show that they are mutually related. In the bipartite case, we give expressions for I-concurrence and negativity, and show that they are also depend on each other. Presented formulas allow to calculate entanglement for an arbitrary size N, while the original definitions practically work for the systems consisting of a few qubits only. We expect that the size dependence of the entanglement of states with elementary excitation may help to understand entanglement in the systems with greater number of elementary excitations.

# Combinatorial approach to the representation of the Schur-Weyl duality in one-dimensional spin systems

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We propose combinatorial approach to the representation of Schur-Weyl duality in physical systems on the example of one-dimensional spin chains. Exploiting the Robinson-Schensted-Knuth algorithm we perform decomposition of the dual groups action into irreducible representations in a fully combinatorial way. As representation space we choose the Hilbert space of the spin chains, but this approach can be easily generalized to an arbitrary physical systems where the Schur-Weyl duality works.

# Yangian Invariants, Unitary Matrix Models and Super Yang-Mills Amplitudes

Nils Kanning

LMU Munich

Joint work with: Yumi Ko, Gregor Richter, Matthias Staudacher

The planar maximally supersymmetric Yang-Mills theory in four-dimensional Minkowski space is believed to be integrable. Its tree-level scattering amplitudes are invariants of a Yangian algebra which is a hallmark of integrability. This symmetry enables very compact expressions for these amplitudes as contour integrals on Grassmannian manifolds. However, these integrals are typically formulated for an unphysical spacetime signature and the choice of the contour is not canonical. We argue that working in Minkowski signature naturally fixes the contour to be a unitary group manifold. We demonstrate the validity of this unitary contour for sample amplitudes. Furthermore, our approach allows to interpret Grassmannian integrals as generalizations of the Brezin-Gross-Witten unitary matrix model. This might make elaborate matrix model technology available for the computation of amplitudes. What is more, our unitary Grassmannian integrals can be used to systematically construct a large class of Yangian invariants beyond scattering amplitudes. Examples of such invariants include solutions of the Yang-Baxter equation that underlie compact and non-compact integrable spin chains.

# Power-spectrum in quantum chaos: Painlevé V solution

Eugene Kanzieper

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Joint work with: Roman Riser (Holon) and Vladimir Al. Osipov (Lund)

We present a non-perturbative analysis of the power-spectrum of energy level fluctuations in fully chaotic quantum structures. Focussing on systems with broken time-reversal symmetry, we employ a finite- $N$  random matrix theory to derive an exact multidimensional integral representation of the power-spectrum. The  $N \rightarrow \infty$  limit of the exact solution furnishes the main result of this study – a universal, parameter-free prediction for the power-spectrum expressed in terms of a fifth Painlevé transcendent. Extensive numerics lends further support to our theory which, as discussed at length, invalidates a traditional assumption that the power-spectrum is merely determined by the spectral form factor of a quantum system.



# Truncation of the orthogonal and symplectic Yangians

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We investigate the structure of truncated Yangians, corresponding to orthogonal and symplectic symmetries. We consider the constraints following from the RLL-relations and the restrictions which they impose on the Yangian representation space.

# Two-species asymmetric diffusion and coupled KPZ equations

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The dynamics of this stochastic process can be described by a master equation with an integrable Hamiltonian which defines a quantum spin chain. This allows us to calculate time-dependent quantities of the non-equilibrium model from the spectrum of the spin chain in equilibrium. We will explain how we used the Bethe Ansatz to calculate the dynamical critical exponent of the underlying stochastic process. In analogy to the single species exclusion process, we define a height model that reflects the nearest-neighbor interactions of the multi-particle exclusion process and derive the partial differential equations for this model. Depending on the parameters of the reaction-diffusion process, the dynamics is of KPZ type, diffusive type or a mixture of both. It is interesting to see that these equations also follow directly from the Master equation approach. This last part is in collaboration with D. Huse and G. Schuetz.

# Quantum Lie theory

V.K. Kharchenko

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The numerous attempts over the previous 15-20 years to define a quantum Lie algebra as an elegant algebraic object with a binary “quantum” Lie bracket have not been widely accepted. In the talk we discuss an alternative approach that includes multivariable operations. There are many fields in which multivariable operations replace the Lie bracket, such as investigations of skew derivations in ring theory, local analytic loop theory, and theoretical research on generalizations of Nambu mechanics. Among the problems discussed in the talk are the following: multilinear quantum Lie operations, the principle generic quantum Lie operation, the basis of symmetric generic operations, Shestakov–Umirbaev operations for the Lie theory of nonassociative products.

# Second order evaluations of orthogonal and symplectic Yangians

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Joint work with: D. Karakhanyan

Orthogonal and symplectic Yangians are defined by the Yang-Baxter relation.  
The conditions of the evaluation of second order are investigated.  
The conditions on the two independent matrices of generators are derived.

# Infinitesimal deformations of Poisson bi-vectors using the Kontsevich graph calculus

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We study a classification problem for universal infinitesimal deformations  $\mathcal{P} \mapsto \mathcal{P} + \varepsilon \mathcal{Q}(\mathcal{P}) + \bar{o}(\varepsilon)$  of Poisson structures  $\mathcal{P}$  on finite-dimensional affine manifolds  $N^n$ . Denote by  $[[\cdot, \cdot]]: \Gamma(\wedge^p TN^n) \times \Gamma(\wedge^q TN^n) \rightarrow \Gamma(\wedge^{p+q-1} TN^n)$  the parity-odd Schouten bracket in the space of multivectors on a manifold  $N^n$  at hand. For all bi-vectors  $\mathcal{P} \in \Gamma(\wedge^2 TN^n)$  such that the Jacobi identity  $\text{Jac}(\mathcal{P}) := [[\mathcal{P}, \mathcal{P}]] = 0$  holds, we seek the Poisson cohomology cocycles  $\mathcal{Q}(\mathcal{P}) \in \Gamma(\wedge^2 TN^n)$  solving  $[[\mathcal{P}, \mathcal{Q}(\mathcal{P})]] \doteq 0$  by virtue of  $[[\mathcal{P}, \mathcal{P}]] = 0$ ; we then quotient out (i) all the coboundaries  $\mathcal{Q} = [[\mathcal{P}, X]]$  given by vector fields  $X \in \Gamma(TN^n)$  and (ii) improper solutions  $\mathcal{Q} = \nabla(\mathcal{P}, \text{Jac}(\mathcal{P}))$  that vanish identically at every Poisson structure  $\mathcal{P}$ . The Kontsevich tetrahedral flow [1,2] is an example of such infinitesimal deformation  $\mathcal{Q}(\mathcal{P})$  which, neither trivial in its cohomology group nor improper, is well defined for all Poisson bi-vectors  $\mathcal{P}$  on all affine manifolds  $N^n$ .

The language of Kontsevich graphs [2] allows to convert this infinite analytic problem within a given set-up  $(N^n, \mathcal{P})$  in dimension  $n$  into a set of finite combinatorial problems whose solutions are universal for all Poisson geometries in all dimensions  $n < \infty$ . The graph realisation of bi-vector  $\mathcal{P}$  suggests a  $k$ -vertex ansatz for the flows  $\mathcal{Q}(\mathcal{P})$  of nonlinearity degree  $k$  at every  $k > 0$ ; the equations

$$\begin{aligned} [[\mathcal{P}, \mathcal{Q}(\mathcal{P})]] &= \diamond(\mathcal{P}, \text{Jac}(\mathcal{P})) && \text{(cocycle),} \\ \mathcal{Q}(\mathcal{P}) &= [[\mathcal{P}, X]] + \nabla(\mathcal{P}, \text{Jac}(\mathcal{P})) && \text{(to quotient out)} \end{aligned}$$

are solved for graphs that encode  $\mathcal{Q}$  and  $\diamond$  and  $\mathcal{Q}$ ,  $X$ , and  $\nabla$ , respectively. For example, the scaling  $\dot{\mathcal{P}} = \mathcal{P}$  is a linear ( $k = 1$ ) nontrivial proper flow on the space of Poisson bi-vectors. Likewise, the Kontsevich tetrahedral flow [1,2] is quartic-nonlinear in  $\mathcal{P}$  and of differential order three, which is because the tetrahedra to encode  $\mathcal{Q}(\mathcal{P})$  contain  $k = 4$  internal vertices inhabited by a copy of the Poisson structure  $\mathcal{P}$  and because three oriented edges come into one vertex in a graph in this solution  $\mathcal{Q}(\mathcal{P})$ .

We report a few-vertex classification (from  $k \leq 4$  towards  $k \lesssim 7$ ) of graph solutions to the universal deformation problem  $\dot{\mathcal{P}} = \mathcal{Q}(\mathcal{P})$  for Poisson structures.

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# Schubert calculus and invariant trilinear forms

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Joint work with: Paul Zinn-Justin

The Knutson-Tao “puzzle” rule for computing equivariant Schubert calculus of Grassmannians was interpreted in [Zinn-Justin ’08] as an integrable model, and extended in [ZJ-Wheeler ’16] to  $K$ -theory. We do the same to give positive rules for Schubert calculus beyond Grassmannians (1-step flag manifolds) in two previously unsolved cases:  $K$ -theory (and equivariant  $K$ -theory) of 2-step flag manifolds (which includes, in particular, equivariant quantum  $K$ -theory of Grassmannians), and cohomology and  $K$ -theory of 3-step flag manifolds.

The nonequivariant rules are based on triangular tiles; correspondingly, in the integrable model there is a special value of the spectral parameter at which the  $R$ -matrix factors through an irrep, breaking the quadrangle into two triangles.

# Radical splitting in associative conformal algebras with finite faithful representation

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Conformal algebras naturally appear as algebraic structures describing singular part of the operator product expansion (OPE) in conformal field theory [1]. From the algebraic point of view, conformal algebra is a linear space  $C$  equipped with a linear map  $\partial : C \rightarrow C$  and binary operation ( $\lambda$ -product)  $(\cdot)_\lambda \cdot : C \otimes C \rightarrow C[\lambda]$ , where  $\lambda$  is a formal variable, such that

$$(\partial a)_\lambda b = -\lambda(a)_\lambda b, \quad (a)_\lambda \partial b = (\lambda + \partial)(a)_\lambda b.$$

Conformal algebra  $C$  is said to be associative if, in addition, the following identity holds for all  $a, b, c \in C$ :

$$(a)_\lambda (b)_\mu c = ((a)_\lambda b)_{\lambda+\mu} c \in C[\lambda, \mu].$$

As an example, consider the matrix algebra  $C = M_n(\mathbb{C}[\partial, x])$  equipped with  $\lambda$ -product

$$A(\partial, x)_\lambda B(\partial, x) = A(-\lambda, x)B(\partial + \lambda, x + \lambda).$$

This is an associative conformal algebra denoted  $\text{Cend}_n$  [2].

If a conformal algebra  $C$  embeds into  $\text{Cend}_n$  for some  $n$  then  $C$  is said to be an associative conformal algebra with finite faithful representation (*f.f.r.*-conformal algebra).

Every f.f.r.-conformal algebra  $C$  has a nilpotent radical  $R$  such that  $A = C/R$  is a semisimple f.f.r.-conformal algebra; all simple and semisimple f.f.r.-conformal algebras were described in [3]. In [4,5,6], the following question was studied: whether the exact sequence of f.f.r.-conformal algebras

$$0 \rightarrow R \rightarrow C \rightarrow A \rightarrow 0 \quad (*)$$

splits, i.e., whether the conformal analogue of the Wedderburn Principal Theorem holds in this class of algebras.

In this work (joint with R. A. Kozlov), we apply cohomological approach to describe completely all those semisimple f.f.r.-conformal algebras  $A$  for which every sequence  $(*)$  splits.

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# Quantum Models Based on Finite Groups

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The trajectory of a quantum system is a sequence of unitary evolutions of vectors in a Hilbert space, interspersed with observations — projections of the vectors in some subspaces, that are specified by measuring devices. Quantum-mechanical description can be made constructive, if we replace the general group of unitary transformations of the Hilbert space by unitary representations of finite groups. It is known that any linear representation of a finite group can be realized as a subrepresentation of some permutation representation. Thus, quantum mechanical problems can be formulated in terms of groups of permutations. Such a constructive approach allows us to clarify the meaning of a number of physical concepts.



# Theoretical study of the neon trimer

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This work is aimed at a theoretical investigation of the neon atomic clusters. Bound state energies and wave functions have been calculated. The modern realistic potential models are used for description the interatomic interaction. Differential Faddeev equations in the total angular momentum representation are used for the binding energies calculations of neon trimer system. The results obtained are compared with previous published results.

# Quantum K-theory of Quiver Varieties and Many-Body Systems

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Joint work with: P. Pushkar, A. Smirnov, A. Zeitlin

We define quantum equivariant  $K$ -theory of Nakajima quiver varieties. We discuss type  $A$  in detail as well as its connections with quantum XXZ spin chains and trigonometric Ruijsenaars-Schneider models. Finally we study a limit which produces a  $K$ -theoretic version of results of Givental and Kim, connecting quantum geometry of flag varieties and Toda lattice. On the way we discover a duality between the five-vertex model and  $q$ -Toda system

# Magnetic monopoles in noncommutative quantum mechanics

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Joint work with: Peter Prešnajder

We discuss certain generalization of the Hilbert space of states in noncommutative quantum mechanics that, as we show, introduces magnetic monopoles into the theory. Such generalization arises very naturally in the considered model, but can be easily reproduced in ordinary quantum mechanics as well. This approach offers a different viewpoint on the Dirac quantization condition and other important relations for magnetic monopoles. We focus mostly on the kinematic structure of the theory, but investigate also a dynamical problem (with the Coulomb potential).

# Oscillators from nonlinear realizations

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Joint work with: Sergey Krivonos

With the use of the method of nonlinear realizations, we prove that for a great number of noncompact Lie algebras it is possible to construct invariant systems of oscillators. All these systems enjoy the conformal symmetry. Their equations of motion can always be made linear by appropriate change of variables in the coset space. These equations of motion are those of harmonic and Pais-Uhlenbeck oscillators with related frequencies, depending on the grading of the Lie algebra. Additionally, we show that it is possible to construct the actions for these systems, with help of the Cartan forms in the extended coset space.

# $G_2$ invariant (deformed)oscillators

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We associate with each simple Lie algebra the system of the second order differential equations possessing this symmetry. In the special limits, these equations reduced to the system of ordinary harmonic oscillators. We provide the clarifying example: systems of (deformed)oscillators invariant with respect to  $G_2$  symmetry. The important question of existence of the corresponding action is analyzed and for the considered case the proper algorithm for construction of the invariant actions is proposed.

# Integrable properties of minimal surfaces in hyperbolic space

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Minimal area surfaces in hyperbolic space (AdS) play a central important role in the AdS/CFT correspondence since they are related to Wilson loops. In this talk I will describe its integrability properties and new methods to find solutions including a numerical approach based on integrability.

# Black hole, hidden symmetries, and complete integrability

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Joint work with: Valeri P. Frolov, Pavel Krtouš

I will overview the recent progress on understanding the remarkable mathematical properties of higher-dimensional rotating black holes. These stem from a single object in the Killing-Yano family of symmetries called the principal tensor. I will show that the principal tensor not only determines the form of the black hole geometry, but also guarantees its special properties, as special algebraic type, complete integrability of geodesic motion, and the existence of symmetry operators for the Hamilton–Jacobi, Klein–Gordon, and Dirac equations.

# Quantization of non-geometric M-theory backgrounds

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Joint work with: R.J. Szabo

We describe the quantization of a four-dimensional locally non-geometric M-theory background dual to a twisted three-torus by deriving a phase space star product for deformation quantization of quasi-Poisson brackets related to the nonassociative algebra of octonions. The construction is based on a choice of G2-structure which defines a nonassociative deformation of the addition law on the seven-dimensional vector space of Fourier momenta. We demonstrate explicitly that this star product reduces to that of the three-dimensional parabolic constant R-flux model in the contraction of M-theory to string theory.



# Color Lie (super) symmetries in Lévy-Leblond equation

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We investigate systems with color Lie (super) algebra symmetries. As an example we consider the symmetries of the Levy-Leblond equation, which is a non-relativistic wave equation of a spin  $\frac{1}{2}$  particle. It is shown that the equation has two kinds of symmetries. One is given by the super Schroedinger algebra and the other one by a  $Z_2 \times Z_2$  graded Lie superalgebra. This structure accommodate all the symmetry operators of the equation and it is a simple example of the  $Z_2 \times Z_2$  symmetries of an equation where all graded subspaces are not empty.

# Galois symmetry of rigged strings and Baxter TQ equation in the three-magnon sector of magnetic heptagonal ring

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We extend our previous results on parity symmetry of the qubit of highest weight states in the three-magnon sector of magnetic heptagonal ring ( $N = 7$  nodes with the spin  $1/2$ ) at the centre of the Brillouin zone (with quasimomentum  $k = 0$ ), by discussion of Galois symmetries of the associated Bethe parameters (portions of phase  $a = \exp(ip)$ , with pseudomomentum  $p$ ), and some conclusions related to famous Baxter  $TQ$  equation for this case. Galois theory for a monic polynomial with 6 roots representing Bethe portions of phase is explicitly demonstrated and interpreted within the midst of coordinate Bethe Ansatz. In particular, the rigged string structure is associated with some subgroups of the Galois group of this monic polynomial.

# Recent developments on classical integrable sigma-models

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We review results on the study of integrable sigma-models, such as the principal chiral model and cosets models, in light of the recent developments on their integrable deformations. In the hamiltonian formalism, all of these models are described by so-called cyclotomic  $r/s$  systems with twist function. After reviewing this framework, we describe how it allows to study the symmetries and conservation laws of the models. We illustrate these results on the examples of the principal chiral model and its Yang-Baxter deformation, exhibiting both local and non-local conserved charges of these models.

# Projective State Spaces for Quantum Gravity

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Instead of formulating the states of a Quantum Field Theory (QFT) as density matrices over a single large Hilbert space, it has been proposed by Kijowski to construct them as consistent families of partial density matrices, the latter being defined over small 'building block' Hilbert spaces. In this picture, each small Hilbert space can be physically interpreted as extracting from the full theory specific degrees of freedom (aka. 'coarse-graining' the continuous theory). This allows to reduce the quantization of a classical field theory to the quantization of finite-dimensional sub-systems, while obtaining robust and well-controlled quantum states spaces.

I will explain how this formalism can be applied to background-independent Quantum Gravity, and, in particular, how it supports the development of a discretization scheme which respects the symmetries of General Relativity (invariance under coordinates change, aka. general covariance).

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# Algebraic realization of form factors in integrable models with diagonal $S$ matrices

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Form factors of (quasi)local operators in integrable models with diagonal  $S$  matrices can be obtained as matrix elements of operators, which are expressed in terms of a Heisenberg algebra. This realization essentially simplifies studying properties of the corresponding operators, including reflection properties, resonance identities and compatibility with reductions.

# Classical and quantum semitoric systems on compact manifolds

Yohann Le Floch

Semitoric systems are integrable systems on 4-manifolds with certain good properties, one of them being that their singularities do not possess hyperbolic components. These systems, as well as their quantum counterparts, have attracted a lot of attention in the past decade. In this talk, I will describe two explicit examples of (families of) semitoric systems on compact manifolds, and some of their properties. The first one, due to Sadovskii and Zhilinskii, is defined on  $S^2 \times S^2$  and has been studied in a joint work with Alvaro Pelayo (UC San Diego). The second one is an original example on  $CP^2$ , obtained in a work in progress with Joseph Palmer (Rutgers).

# On Quantum Integrable Models with Intrinsic Degree of Freedoms – our past approaches and recent new result

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In this talk, firstly, I will present the key points of our past approaches on several one-dimensional quantum integrable models that can be realized in experiments. For example, the ground state of  $SU(2)$  bosons [Li,Gu,Ying,Eckern, EPL 61 (2003)368] is a pseudo-spin ferromagnetic state; the Bethe ansatz equation of  $SU(1-2)$  Bose-Fermi mixtures [Hu,Zhang,Li, JPA 39 (2006)351] consists of three-type expressions; Spin model with orbital degeneracy [Li,Ma,Shi,Zhang, PRL 81 (1998)3527; *ibid*, PRB 60(1999)12781] possesses a  $SU(4)$  symmetry and its effects caused by external fields [Gu,Li, PRB 66 (2002) 092404, Gu,Li,Zhou, PRB 69(2004)144405] exhibits very rich phase diagrams; There is a profound relation between entanglement and quantum phase transitions [Gu,Lin,Li, PRA 68(2003)042330]. Then I will show an artificial model [Li, PLA 1996] that is actually also realizable, for example, recently, we consider a Hubbard model in one dimension where there exists both on-site particle-particle interaction and particle-image interaction. We suggested, for two-particle case, an experimental realization of such a seemingly artificial model. We also derived the secular equation of the spectrum for various possible symmetries [Du,Lu,Li, preprints 2017].

# Flat structures and Painlevé equations

Toshiyuki Mano

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In this talk, we study a relationship between regular flat structures and isomonodromic deformations of linear differential equations. The main result is that we can introduce a flat structure on the space of independent variables of the isomonodromic deformation of a generically regular generalized Okubo system. As an application, we show that it is possible to define flat structures on the spaces of independent variables of the classical Painlevé equations (except for P1). In particular, there is a correspondence between solutions to 4-dimensional extended WDVV equation with a particular conditions and generic solutions to the Painlevé equations P2-P6. In this picture, we see that the degeneration scheme of Jordan normal forms of a square matrix of rank four naturally corresponds to the well-known coalescence cascade of the Painlevé equations. This talk is based on joint works with Mitsuo Kato, Jiro Sekiguchi and Hiroshi Kawakami.



# Monodromy in planar potential scattering

Nikolay Martynchuk

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Joint work with: Holger Waalkens

Hamiltonian monodromy was introduced by J.J. Duistermaat as an obstruction to the existence of global action coordinates in bound integrable systems. Since then Hamiltonian monodromy and its quantum counterpart were shown to be present in various physical systems, such as the hydrogen atom (this is a recent result due to H.R. Dullin and H. Waalkens).

In this talk we discuss scattering monodromy, an invariant which naturally appears in certain unbound integrable systems. As opposed to Hamiltonian monodromy, scattering monodromy does not obstruct the existence of global action coordinates; in the quantum case it obstructs the existence of a globally smooth phase shift.

Our main result relates scattering monodromy with Knauf's degree of scattering for systems defined by a central short-range potential on a plane. The long-range case will be also addressed in the talk.

# Ponzano–Regge spin networks and their symmetries

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The 1968 paper 'Semiclassical limit of Racah coefficients' by Giorgio Ponzano and Tullio Regge stands at the basis of a lot of developments and applications in quantum chemistry, low-dimensional geometric topology, quantum computing and discretized gravity models. I will highlight a few algebraic features of the Ponzano-Regge model with an emphasis on the role of Regge symmetries of the  $6j$  symbol and their connections with quadratic operator algebras and projective configurations.

# Description of $(p, q)$ -brane web from $q$ -deformed toroidal symmetry

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We apply Ding-Iohara-Miki algebra to brane-web configuration through Awata-Feigin-Shiraishi intertwiner. In particular, we show how Gaiotto state and related intertwiner are expressed through it. We also define  $q$ -Weyl transformation through which  $qq$ -character can be systematically defined.

# Fractional Statistics phenomena as a Deformation of a Hodge Structure

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Joint work with: Grzegorz Banaszak and Tadeusz Lulek

The fractional statistics parameter for an anyons system on a plane impose an equivariant symmetry on wave functions. They are not univalent ones and the quantum number of angular momentum is non integer either non half integer. The space of quantum states for anyons system, admits a deformed Hodge structure. Especially the space of anyonic harmonic functions admits such a structure. The deformation parameters of the structure are given by fractional statistics. The quantum numbers of angular momenta determine the weights of deformed Hodge structure and they are given by fractional statistics.

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# Charges in nonlinear higher-spin theory

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Joint work with: V.E. Didenko, M.A. Vasiliev

Nonlinear higher-spin equations in four dimensions admit a closed two-form that defines a gauge-invariant global charge as an integral over a two-dimensional cycle. It is argued that this charge gives rise to partitions depending on various lower- and higher-spin chemical potentials identified with modules of topological fields in the theory. The resulting partition is non-zero being in parametric agreement with the ADM-like behavior of a rotating source.

# Spectral Structure of Elastic Neumann–Poincaré Operators

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Joint work with: Kazunori Ando and Hyeonbae Kang

The elastic Neumann–Poincaré (abbreviated by NP) operator for the Lamé operator appears naturally when solving the boundary value problems for the Lamé system. We prove that the elastic NP operator defined on the smooth boundary of a bounded domain in three dimensions, which is known to be non-compact, is in fact polynomially compact. As a consequence, we prove that the spectrum of the elastic NP operator consists of three non-empty sequences of eigenvalues accumulating to certain numbers determined by Lamé parameters. We also mention about related works that deal with important applications.

# Matrix KdV: Tropical limit and Yang-Baxter map

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The tropical limit (Maslov dequantization) of the 2-soliton solution of the matrix KdV equation provides a point particle scattering picture for the resulting Yang-Baxter map. This is based on joint work with Aristophanes Dimakis (University of the Aegean, Greece).

# Symmetries characterizing Borcherds products

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Borcherds products are automorphic forms defined as infinite products. We will give certain symmetries for Siegel modular forms of degree 2, which characterize (holomorphic) Borcherds products.



# Irregular conformal blocks and Painlevé tau functions

Hajime Nagoya

Kanazawa University

We expect that all Painlevé tau functions admit series expansion formulas as Fourier transforms in terms of Virasoro conformal blocks. I explain how we prove the expansion formula of tau functions for PV and PIV and give conjectural formulas of them for PIII and PII.

# Difference-differential Lax representations of fourth and fifth Painlevé equations

Nobutaka Nakazono

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In this talk, we show a reduction of a system of ABS equations to higher order Painlevé equations containing the fourth and fifth Painlevé equations. Using this relation we construct the difference-differential Lax representations of the fourth and fifth Painlevé equations.

# Comparioson of realizations of Lie algebras and their differential invariants

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Joint work with: S. Pošta

Realizations of Lie algebras, their construction and differential invariants are discussed from the equivalence point of view. We propose the practical scheme that allows to define if two fixed realizations are equivalent. Several important examples are shown.

# Quantum variational principle and quantum multiforms

Frank Nijhoff

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Joint work with: S.D. King

A modern notion of integrability is that of multidimensional consistency (MDC) which classically implies the coexistence of commuting dynamical flows in several (possibly infinitely many) variables imposable on one and the same dependent variable. Recently a novel variational principle was introduced to capture this phenomenon within a Lagrangian framework, namely that of Lagrangian multiforms. We will review some of the classical results and propose a quantum analogue of this theory in terms of quantum mechanical propagators.

# Black Hole Quasinormal Modes from Classical Conformal Blocks

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Quasinormal modes are certain complex frequency modes that resonate when a black hole is weakly perturbed. Their determination is important for the study of stability of black holes, to plasma thermalization via AdS/CFT, as well as to the decay of gravitational waves. Here we present a general framework to calculate quasinormal modes through the link of Fuchsian equations, isomonodromic deformations and  $c = 1$  CFT. We also discuss how this picture might be connected to dual CFT descriptions of black holes.

# The component structure of conformal supergravity invariants in six dimensions

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Joint work with: Daniel Butter, Gabriele Tartaglino-Mazzucchelli

Invariants for (super)conformal gravity arise naturally in the study of (super)conformal field theories on curved manifolds, where they span the type B Weyl anomalies. Starting from a superspace formulation, I will present the component structure of the superconformal gravity invariants.

# Quantised relativistic membranes and non-perturbative checks of gauge/gravity duality

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We compare the bosonic and maximally supersymmetric membrane models. We find that in Hoppe regulated form the bosonic membrane is well approximated by massive Gaussian quantum matrix models. In contrast the similarly regulated supersymmetric membrane, which is equivalent to the BFSS model, has a gravity dual description.

# Deformations of the Almheiri-Polchinski model

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We study Yang-Baxter deformations of a specific 1+1 D dilaton gravity model called the Almheiri-Polchinski model. We present a deformed black hole solution and compute the Bekenstein-Hawking entropy. The entropy can also be reproduced holographically by evaluating the renormalized stress tensor on a regularized screen close to a singularity which is generated by the deformation. This work is based on the collaboration arXiv:1701.06340 with Hideki Kyono and Kentaroh Yoshida.



# Quantum theory in real Hilbert space: How the complex Hilbert space structure emerges from Poincaré symmetry

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Joint work with: Valter Moretti

In principle, the lattice of elementary propositions of a generic quantum system admits a representation in real, complex or quaternionic Hilbert spaces as established by Solèr's theorem (1995) closing a long standing problem that can be traced back to von Neumann's mathematical formulation of quantum mechanics. However up to now there are no examples of quantum systems described in real Hilbert spaces. We show that elementary relativistic systems (in Wigner's approach) cannot be described in real Hilbert spaces as a consequence of some peculiarity of continuous unitary projective representations of  $SL(2, \mathbb{C})$  related with the theory of polar decomposition of operators. Indeed such a "naive" attempt leads necessarily to an equivalent formulation on a complex Hilbert space. Although this conclusion seems to give a definitive answer to the real-quantum-mechanics issue, it lacks consistency since it does not derive from more general physical hypotheses as the complex one does. Trying a more solid approach we end up with three possibilities: an equivalent description in terms of a Wigner unitary representation in a real, complex or quaternionic Hilbert space. At this point the "naive" result turns out to be a definitely important technical lemma, for it forbids the two extreme possibilities. In conclusion, the real theory is actually complex. [Based on the paper arXiv:1611.09029]

# From free bosons to 2D TQFT

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Joint work with: Christian Korff

We define a family of 2D TQFT in terms of free bosons and show that each TQFT is a generalised Verlinde algebra: there exists a representation of the modular group and the TQFT fusion coefficients are given by the Verlinde formula. We show that the fusion coefficients are non-negative integers by providing different combinatorial expressions, and that they are related to tensor product multiplicities of irreducible representations of the generalised symmetric group. Finally, we will relate the fusion coefficients of different 2D TQFTs via recursion formulae, taking advantage of the Heisenberg algebra. Our construction is motivated by a recent definition of a 2D TQFT in terms of the  $q$ -boson model of which this is the  $q = 1$  or classical limit. At  $q = 0$  one obtains the familiar  $\mathfrak{su}(n)$  Verlinde algebra of WZW conformal field theory.

# Szegő kernels and asymptotic expansions for Legendre polynomials

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We present a geometric approach to the asymptotics of the Legendre polynomials  $P_{k,n+1}$ , based on the Szegő kernel of the Fermat quadric hypersurface, and leading to complete asymptotic expansions holding on expanding subintervals of  $[-1, 1]$ .

# Super Yangian of the general linear Lie superalgebra and 01-sequence

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In this talk, we will study the super Yangian  $Y_{M|N}$  associated to the general linear Lie superalgebra  $\mathfrak{gl}_{M|N}$  by introducing the notion of 01-sequence. In particular, a series of new presentations of  $Y_{M|N}$  is obtained. Some further applications will be discussed.

# Boundary conditions for General Relativity on $\text{AdS}_3$ and the KdV hierarchy

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Joint work with: David Tempo and Ricardo Troncoso

It is shown that General Relativity with negative cosmological constant in 3D admits a new family of boundary conditions being labeled by a nonnegative integer  $k$ . Gravitational excitations are then described by “boundary gravitons” that fulfill the equations of the  $k$ -th element of the KdV hierarchy. In particular,  $k = 0$  corresponds to the Brown-Henneaux boundary conditions so that excitations are described by chiral movers. In the case of  $k = 1$ , the boundary gravitons fulfill the KdV equation and the asymptotic symmetry algebra turns out to be infinite-dimensional, abelian and devoid of central extensions. The latter feature also holds for the remaining cases that describe the hierarchy ( $k > 1$ ). Our boundary conditions then provide a gravitational dual of two noninteracting left and right KdV movers, and hence, boundary gravitons possess anisotropic Lifshitz scaling with dynamical exponent  $z = 2k + 1$ . Remarkably, despite spacetimes solving the field equations are locally AdS, they possess anisotropic scaling being induced by the choice of boundary conditions. As an application, the entropy of a rotating BTZ black hole is precisely recovered from a suitable generalization of the Cardy formula that is compatible with the anisotropic scaling of the chiral KdV movers at the boundary. The extension of our boundary conditions to the case of higher spin gravity and its link with different classes of integrable systems is also briefly discussed.

# Transfer matrix spectrum for cyclic representations of the 6-vertex reflection algebra

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Joint work with: J.-M. Maillet, G. Niccoli

The transfer matrix spectral problem for the cyclic representations of the trigonometric 6-vertex reflection algebra associated to the Bazanov-Stroganov Lax operator is studied. This contains, as a particular case, the spectral analysis of the lattice sine-Gordon model with open boundary conditions. The method of separation of variables (SoV) is used to perform the spectral analysis: both the eigenvalues and eigenstates are completely characterized in two ways. First, in terms of the set of solutions to a discrete system of polynomial equations in a given class of functions, and then as the set of solutions to a Baxter's like  $T$ - $Q$  functional equation, allowing to rewrite the transfer matrix eigenstates in an algebraic Bethe ansatz form.

# Solvable spectral problems from 2d CFT and $\mathcal{N} = 2$ gauge theories

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The so-called 2d/4d correspondences connect two-dimensional conformal field theory (2d CFT),  $\mathcal{N} = 2$  supersymmetric gauge theories and quantum integrable systems. The latter in the simplest case of  $SU(2)$  gauge group are quantum-mechanical systems. During a talk it will be shown how these dualities can be used, in particular, to study spectra of Schroedinger operators with periodic (Hill's),  $PT$ -symmetric and elliptic (finite-gap) potentials. Moreover, it will be mentioned that an application of 2d CFT technics and tools such as the (classical) conformal bootstrap paves the way for studying nonperturbative effects in aforementioned quantum-mechanical systems. Finally, some open problems such as the relation to the KdV theory will be mentioned as well.

# New separated polynomial solutions to the Zernike system on the unit disk and interbasis expansion

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Joint work with: Kurt Bernardo Wolf and Alexander Yakhno

The differential equation proposed by Frits Zernike to obtain a basis of polynomial orthogonal solutions on the unit disk, is shown to have a set of new orthonormal solution bases, involving Legendre and Gegenbauer polynomials, in non-orthogonal coordinates close to Cartesian ones. The operators which provide the separation constants are found to participate in a superintegrable cubic Higgs algebra. We find the interbasis expansions between the original Zernike basis and a representative of the new set, which turn out to be Clebsch-Gordan coefficients.



# On finite $W$ -algebras for Lie algebras and superalgebras

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Joint work with: V. Serganova

A finite  $W$ -algebra is a certain associative algebra attached to a pair  $(\mathfrak{g}, e)$ , where  $\mathfrak{g}$  is a complex semisimple Lie algebra and  $e \in \mathfrak{g}$  is a nilpotent element. E. Ragoucy and P. Sorba described finite  $W$ -algebras for  $\mathfrak{gl}(n)$  when the matrix  $e$  has Jordan blocks of the same size. J. Brundan and A. Kleshchev generalized their result to an arbitrary nilpotent element. We study finite  $W$ -algebras for the queer Lie superalgebra  $Q(n)$  associated with non-regular even nilpotent coadjoint orbits, in the case when the corresponding nilpotent element has Jordan blocks of the same size  $l$ . We prove that this finite  $W$ -algebra is isomorphic to a quotient of the super-Yangian of  $Q(\frac{n}{l})$ .

# Involutive representations of coordinate algebras and quantum spaces

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Joint work with: T.Jurić and J.-C.Wallet

We show that  $\mathfrak{su}(2)$  Lie algebras of coordinate operators related to quantum spaces with  $\mathfrak{su}(2)$  noncommutativity can be conveniently represented by  $SO(3)$ -covariant poly-differential involutive representations. We show that the quantized plane waves obtained from the quantization map action on the usual exponential functions are determined by polar decomposition of operators combined with constraint stemming from the Wigner theorem for  $SU(2)$ . Selecting a subfamily of  $*$ -representations, we show that the resulting star-product is equivalent to the Kontsevich product for the Poisson manifold dual to the finite dimensional Lie algebra  $\mathfrak{su}(2)$ . We discuss the results, indicating a way to extend the construction to any semi-simple non simply connected Lie group and present noncommutative scalar field theories which are free from perturbative UV/IR mixing.

# Magnetic monopoles in fuzzy space and non-associativity

Petr Presnajder

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We investigate quantum mechanics in fuzzy space with wave functions in the Hilbert space that induces magnetic monopoles in the free theory: The commutator of velocities equals to fuzzy generalization of the magnetic monopole field strength. The resulting symmetry is a fuzzy deformation of the usual non-associative symmetry of the magnetic monopole. We discuss a relativistic non-associative model based on analogous structures.

# Exact solutions for the nonlinear Luttinger liquid from Painleve functions and integrable classical PDEs

Tom Price

Utrecht University

Joint work with: Dmitry Kovrizhin, Austen Lamacraft

I will talk about exact solutions for the Green function of Imambekov-Glazman's nonlinear Luttinger liquid theory in terms of classically integrable differential equations. The central result is that at zero temperature the time dependent Green function takes a scaling form and is the tau function for the fourth Painleve transcendent, which may be solved numerically to high accuracy. I will also discuss the extension to finite temperatures.

# On free field realization of the twisted Heisenberg–Virasoro algebra at level zero and $W$ –algebra $W(2,2)$

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Joint work with: Dražen Adamović

Realization of highest weight modules of the twisted Heisenberg–Virasoro algebra at level zero  $\mathcal{H}$  is obtained by vertex algebraic methods. In particular, Heisenberg–Virasoro VOA is realized as kernel of a screening operator inside of Heisenberg VOA. Then one can obtain highest weight modules by means of VOA associated to certain hyperbolic lattice. Furthermore,  $W(2,2)$  VOA is realized as kernel of a screening operator inside of  $\mathcal{H}$  VOA. In this way the highest weight  $\mathcal{H}$ –modules become  $W(2,2)$ –modules and we can show which irreducible  $\mathcal{H}$ –modules are also irreducible over  $W(2,2)$ .

Supported by Croatian Science Foundation project 2634 and Croatian Scientific Centre of Excellence QuantiXLie.

# Integrals of motion and trajectories for the Perlick system I: an algebraic approach

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Joint work with: S.Kuru, J.Negro

The Perlick I type system is a one-parameter family of maximally superintegrable systems complying with the requirements of Bertrand's Theorem. Accordingly, for the classical model there exist stable circular orbits and all all bounded trajectories are closed. In this talk, we focus our attention on the symmetry algebra of its integrals of motion, that is constructed through the so-called "*factorization method*", yielding both "*shift*" and "*ladder*" functions. The knowledge of the symmetry algebra leads in a natural way to identifying the orbit equations. A thorough illustration of both open and closed trajectories is given. Possibly interesting future developments are briefly outlined.

# Calculating algebraic entropies: an express method

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Joint work with: B. Grammaticos, R. Willox, T. Mase

We describe a method for investigating the integrable character of a given three-point mapping, provided that the mapping has confined singularities. Our method, dubbed “express”, is inspired by a novel approach recently proposed by R.G. Halburd. While the latter aims at computing the exact degree growth of a given mapping based on the structure of its singularities, we content ourselves with obtaining an answer as to whether a given system is integrable or not. We present several examples illustrating our method as well as its limitations.

# Vertex Algebras at the Corner

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Starting from the  $U(N)$  Kac-Moody vertex operator algebra, there exist three ways of producing  $W_N$  algebra using coset construction and two Drinfeld-Sokolov reductions at different levels. We generalize these constructions to produce a four parameter family of algebras  $Y_{L,M,N}[\Psi]$  that can be identified with algebras of operators living at junctions of domain-walls in topologically twisted  $\mathcal{N} = 4$  super Yang-Mills theory. S-duality action on the system of defects induces triality action on  $Y$ -algebras, generalizing duality between the three realizations of  $W_N$ . We comment on huge potential for generalizations to algebras associated to more complicated configurations of domain-walls, relation of  $Y$ -algebras to truncations of  $W_\infty$  and relation to the counting of D0-D2-D4 bound states.



# Conformal Symmetry of a 2D Coulomb gas

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Joint work with: Seung-Yeop Lee (University of South Florida, Tampa)

We consider the set of  $n$  point particles in the complex plane that interact via two-dimensional (i.e. logarithmic) Coulomb repulsion and are subject to a confining potential with some growth condition at infinity. It is well known that in the limit when  $n$  goes to infinity, the particles fill a compact domain  $K \subset \mathbb{C}$ . We restrict to the cases where the potential is a non-critical Hele-Shaw potential and  $K$  is simply connected. For the Hele-Shaw potential the averaged density of particles becomes constant inside  $K$ . Of special interest is the density function at the boundary (in rescaled coordinates holding the mean distance among the particles constant). We will present leading and subleading asymptotics. The conformal map from the outside of  $K$  to the outside of the unit disk encodes the geometry of  $K$ . We will show a universality law depending on this conformal map. Further we will discuss the behavior of the correlation among the particles.

# A Gauge Theory Formulation for Continuous and Higher Spin Particles

Victor Rivelles

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Continuous spin particles are one of the irreducible representations of the Poincare group. Its properties are largely unknown since standard gauge theories techniques have been shown to be problematic. Recently a gauge theory formulation on a cotangent bundle was found. We will discuss its properties and shortcomings.

# Generalized Pareto optimum and semi-classical spinors; applications to the graphene Hamiltonian

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In 1971, S. Smale presented a generalization of Pareto optimum he called the critical Pareto set. The underlying motivation was to extend Morse theory to several functions, i.e. to find a Morse theory for  $m$  differentiable functions defined on a manifold  $M$  of dimension  $\ell$ . Here we propose to extend this framework to a  $2 \times 2$  Hamiltonian  $H = H(p) \in C^\infty(T^*\mathbf{R}^2)$  for spinors (including the Graphene Hamiltonian or an Hamiltonian of the type used in elasticity theory) in the sense that we find conditions such that  $H(p) - E$  decomposes locally, up to a conjugation with regular matrices, as  $(u'_+)C_+(u'_+)^* - (u'_-)C_-(u'_-)^*$ , where  $u : \mathbf{R}^2 \rightarrow \mathbf{R}^2$  has singularities of codimension 1 or 2, and  $C_\pm$  are Hermitian regular matrices (“integrating factors”). For instance, when  $A = \frac{1}{2}$ , the Hamiltonian  $H = \begin{pmatrix} \xi^2 + A\eta^2 & A\xi\eta \\ A\xi\eta & A\xi^2 + \eta^2 \end{pmatrix}$  used in elasticity theory with Dirac point at  $(\xi, \eta) = 0$ , provides an integrable system with quadratic integral, in the sense that  $H = u'Cu^*$  with  $u(\xi, \eta) = (2^{-5/4}(\xi^2 + \eta^2), 2^{-1/4}\xi\eta)$ , and  $C = \text{diag}(2^{1/2}, 2^{-1/2})$ . More generally, we can investigate a normal form for the Graphene Hamiltonian near a Dirac point (codimension 2 singularity). This decomposition allows in turn to construct approximate wave-functions near such a point in the semi-classical limit.

# Principal subspaces of twisted modules for certain lattice vertex operator algebras

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Joint work with: Michael Penn

Principal Subspaces of standard modules for affine Lie algebras were originally defined and studied by Feigin and Stoyanovsky, and have been studied by many authors since. In this talk, we discuss how vertex-algebraic techniques can be used to study principal subspaces of basic modules for twisted affine Lie algebras. We also discuss how these techniques can be extended to the study of principal subspaces of twisted modules for certain lattice vertex operator algebras. In particular, we extend earlier work by Capparelli, Calinescu, Lepowsky, and Milas to these settings, and derive exact sequences and recursions satisfied by the multigrade dimensions of the principal subspaces of interest. Solving these recursions, we obtain the multigrade dimensions of these principal subspaces.

# The Arcsine law, Quantum-Classical Correspondence and Orthogonal polynomials

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From the viewpoint of quantum probability (noncommutative probability), we will show that the Arcsine law appears universally in the mathematical structure of quantum-classical correspondence and in the asymptotic behavior of orthogonal polynomials.

# A rigid irreducible Fuchsian $q$ -difference equation can be reduced to a 1st order equation by integral transformation

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We construct a  $q$ -analog of middle convolution. We can reduce the rank of the system by using this, when the system is irreducible, Fuchsian, and of rigidity index 2. In the terms of the middle convolution, we can consider a classification theory of Fuchsian  $q$ -difference equations.

# A note on graviton exchange in emergent gravity scenario

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Joint work with: Hikaru Kawai and Kiyoharu Kawana

We investigate the noncommutative(NC) interpretation of the matrix model, which is believed to contain gravity. We analyze whether the scattering amplitude for graviton exchange is reproduced through the NC gauge field theory from the matrix model. There it is suggested that the NC parameter should be treated as a dynamical variable in order to compensate the off-shell DOF of gravitons.

# Yang-Baxter sigma models, conformal twists and noncommutative Yang-Mills

Junichi Sakamoto

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The Yang-Baxter sigma model is a systematic way to generate integrable deformations of  $\text{AdS}_5 \times S^5$ . In this talk, we recast the deformations as seen by open strings, where the metric is undeformed  $\text{AdS}_5 \times S^5$  with constant string coupling, and all information about the deformation is encoded in the noncommutative (NC) parameter. We identify the deformations of  $\text{AdS}_5$  as twists of the conformal algebra, thus explaining the non-commutativity. In addition, we show that the unimodularity condition on  $r$ -matrices for supergravity solutions translates into the NC parameter being divergence-free. This talk is based on arXiv:1702.02861



# $\Phi^3$ model – from a matrix model to a field theory

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We solve the  $\Phi^3$  matrix models in a limit which are regarded as 2,4 and 6-dimensional theories. The matrix models are known as regularised Kontsevich models. We show how Ward-Takahashi identities and Schwinger-Dyson equations lead in a special large- $\mathcal{N}$  limit to integral equations that we solve exactly for all correlation functions. The solved models arises from noncommutative field theories in a special limit of strong deformation parameter.

# On low-energy effective action in 2d SQED

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We study low-energy effective action in two-dimensional supersymmetric electrodynamics in Coulomb branch. In  $\mathcal{N} = (2, 2)$  superspace, this effective action is described by Kähler potential for twisted chiral superfield. We compute two-loop quantum correction to this potential and determine the corresponding sigma-model metric. Some features of low-energy effective action in  $\mathcal{N} = (4, 4)$  SQED are also discussed.

# Highlighting mechanism of symmetric configurations in random tensor networks and canonical tensor model

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We discuss a mechanism by which group-symmetric configurations are highlighted due to quantum coherence. After general arguments, we specifically discuss the outcome of the mechanism in randomly connected tensor networks and canonical tensor model. The mechanism may provide a way of generating spacetimes with global symmetries as well as other symmetries existing in nature, which are usually postulated.

# (Super-)symmetries of the quantum planar pendulum in different coordinate systems

Simon Becker<sup>(1)</sup>, Marjan Mirahmadi<sup>(1)</sup>, Burkhard Schmidt<sup>(1)</sup>,  
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We apply the quantum Hamilton-Jacobi formalism to the planar pendulum eigenproblem for different choices of coordinates (polar, logarithmic, and hyperbolic) and derive the corresponding conditions of quasi-exact solvability as well as the exact solutions. In each case we identify the topological index  $\kappa$  which determines the loci of the intersections (genuine and avoided) of the eigenenergy surfaces spanned by the dimensionless interaction parameters  $\eta$  and  $\zeta$  and examine the types of ( $N$ -fold) supersymmetry in relation to the spatial symmetries of the pendulum system.

# **Solitons of the generalized coupled Schrodinger-Maxwell-Bloch system**

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We present the system of generalized coupled Schrodinger-Maxwell-Bloch equations which governs the propagation of optical pulses in nonlinear fiber. The Lax pair is explicitly constructed and the soliton solution is obtained using the Darboux transformation.

# Stokes Wedges in WKB and quantum field controlling factor methods for the class $-(ix)^n$ of $\mathcal{PT}$ -symmetric Theories

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In the study of  $\mathcal{PT}$ -symmetric theories, the Stokes wedges are generated in QM using WKB prediction for the wave function that are localized in regions in the complex  $x$ -plane that satisfies the condition. In QFT, they are generated using the controlling factor  $\int d\phi \exp(-V(\phi))$ , where the integral runs over a complex contour  $\Gamma$  in the complex  $\phi$ -plane and has to exist as well. In 0+1 space-time dimensions quantum field and QM coincides. In this work, we generate the Stokes wedges for the class,  $-(ix)^n$  using both methods and show that the results are different for the same theory. The most important realization is that while half of the Hamiltonians in the class are real line theories in QM treatment, a larger number of Hamiltonians are non-real line theories in QF approach. We conjecture the reason behind this discrepancy as well as explain which results are to be followed in a study of PT-symmetric theories.

# Conformal Amplitude Hierarchies and the Poincaré Disk

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We study the amplitudes of intermediate channels in the 4-point function of the fundamental fields in the conformal field theory (CFT) of the  $O(n)$  model, whose positivity has recently played a crucial role in the conformal bootstrap for  $n > 0$  in dimensions  $d > 2$ . For  $d = 2$ , the analytic properties of the amplitudes may be seen from a certain hierarchical structure of the piecewise geodesics if the complex parameter  $n$  is mapped onto the Poincaré disk, which naturally incorporates the symmetry of the congruence subgroup  $\Gamma(2) \subset SL(2, \mathbb{Z})$ . For each channel, we have identified all the possible zeros and poles along the boundary of the disk where CFTs are realized ( $n \in \mathbb{R}$ ), which result in two dense series of the finite operator product expansions (OPEs) and the logarithmic ones, respectively. We show the finite OPE can be embedded into the celebrated minimal model associated with the quantum group  $U_q(sl_2)$  for  $q$  at a root of unity, while the logarithmic OPE is realized by highly non-trivial cancellations between the regular and the singular conformal blocks, and thus is closely related to the positivity (unitarity) violation.

# Stable and Unstable Periodic Orbits in the Anisotropic Kepler Problem (AKP)

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Joint work with: Keita Sumiya (FGMS, Komazawa Univ.) and Kazuhiro Kubo (Dept. of Phys., Bar Iran Univ.)

The Anisotropic Kepler Problem (AKP) connects its integrable limit to its ergodic domain just with one parameter—the mass anisotropy. Especially two-dimensional AKP (AKP2) is mathematically interesting Hamiltonian system admitting a binary coding of orbits. It has been proved for AKP2 that there is at least one periodic orbit (PO) for any given code if the mass anisotropy exceeds a given bound.

We challenge the unsolved converse problem—the uniqueness of the PO for a given code—and give a resolution as follows. Consider a level  $N$  surface over the initial domain  $D$  whose height at a  $p \in D$  is calculated by the first  $N$  bits of the orbit starting from  $p$ . The surface consists of  $2^{N+1}$  steps. Based on the structure of one-time map governed by hyperbolic singularities we prove that the surface is monotonically increasing by mathematical induction with respect to  $N$ . In general, the steps become finer with increasing  $N$  and the initial value of the PO is singled out as a point in  $D$  at  $N \rightarrow \infty$ . This leads to the uniqueness of the PO. However, we also show that there are special codes for certain anisotropies whose the steps do not shrink at large  $N$ . Then not only an isolated unstable PO but also its partner stable PO of the same code bifurcates. This settles the uniqueness issue in AKP2. We show that it is possible to extract from seemingly random energy spectrum data of AKP both periodicities and Lyapunov exponents of various POs.



# Numerical solution of the radial Loewner equation

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Joint work with: Ikkei Hotta (Yamaguchi University)

The radial Loewner partial differential equation provides a one-parametric family of conformal maps on the unit disk whose images describe a flow of an expanding simply-connected domain on the complex plane. In this talk, we will introduce a numerical method for solving the radial Loewner equation. Properties of approximants and numerical experiments will be presented.

# Deformed $\mathcal{N}=8$ supersymmetric mechanics

Stepan Sidorov

Joint Institute for Nuclear Research, Dubna  
Moscow Region, 141980 Russia

Joint work with: Evgeny Ivanov, Olaf Lechtenfeld

We introduce a new kind of non-relativistic  $\mathcal{N}=8$  supersymmetric mechanics, associated with worldline realizations of the supergroups  $SU(2|2)$  and  $SU(4|1)$  treated as deformations of the flat  $\mathcal{N}=8$ ,  $d=1$  supersymmetry. Common features are mass oscillator-type terms proportional to the deformation parameter  $m$  and trigonometric realizations of superconformal groups in the conformal cases.

# CalogeroMoser spin systems for the cyclic quiver

A. Silantyev

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Joliot-Curie, Dubna, Russia

Joint work with: O. Chalykh

The CalogeroMoser spaces (for type  $A_n$ ) arose in application to the KP hierarchy [Wilson]. These are some completions of the symmetrized phase spaces of the rational Calogero-Moser systems (for any complex reflection group). For the generalized symmetric group  $S_n \times \mathbb{Z}_m^n$  the CalogeroMoser space can be obtained from the cyclic quiver  $Q$  with  $m$  vertices [Etingof-Ginzburg]: types  $A_n$  and  $B_n$  corresponds to  $m = 1$  and  $m = 2$ . This space is the set of isomorphism classes of representations of the (deformed) preprojective algebra for the quiver  $Q$  with the simplest non-trivial framing. We considered more general framings in application to generalized (matrix) KP hierarchy and, as a by-product, we obtained new types of integrable systems: CalogeroMoser systems with spin-like variables, which generalize, on the other hand, the GibbonsHermsen spin system.

# Color quasi-hom-Lie-algebras and quasi-hom-Lie quasi-deformations of Witt and Virasoro algebras

Sergei Silvestrov

Division of Applied Mathematics,  
Mälardalen University  
Box 833, Västerås  
Sweden

In this talk I will give a brief review on color Quasi-hom-Lie-algebras and construction of color quasi-deformations of Witt and Virasoro type algebras of vector fields.

# Integration of Courant algebroids

Michal Siran

Faculty of Mathematics, Physics and Informatics, Comenius University Bratislava  
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Slovakia

A Courant algebroid can be thought of as a symplectic dg manifold of degree 2. By proving Lie's Third Theorem for  $L_\infty$ -algebroids (dg manifolds) we can integrate the underlying dg manifold of a Courant algebroid. Following the AKSZ construction, one can then integrate the symplectic structure. The resulting integral is a local 2-symplectic Lie 2-groupoid.

# Dynamical generation of fermion mixing

Luca Smaldone

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Via Giovanni Paolo II, 132 84084 Fisciano, Italy

Joint work with: Massimo Blasone, Petr Jizba

A careful non perturbative study of flavor mixing reveals an interesting structure of the flavor vacuum. This is deeply related to the existence of unitarily inequivalent representations of field algebra in Quantum Field Theory. The particle-antiparticle condensate structure of the flavor vacuum suggests the idea of fermion mixing as an emergent dynamical phenomenon. An attempt in this direction, using operator formalism, was recently formulated, and gap equations were obtained both for the masses and mixing angles. If we try to derive these sets of gap equations by using one-loop effective action with the help of path integral techniques, the analysis of this problem evokes two immediate questions: i) Does path integral know about inequivalent representations? ii) Is it the standard generating functional of Green's functions capable of distinguishing among different inequivalent vacua? The answer to the first question is positive, as can be proved within the coherent state functional integrals framework. However, the answer to the second question opens new interesting scenarios, leading to the necessity of a generalization of the standard definition of the generating functional of Green's functions, in order to take into account the rich vacuum structure of QFT.

# Graphene and phosphorene nanoribbons in magnetic field

Jan Smotlacha

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Joint work with: Richard Pincak

The electronic properties of the zigzag nanoribbons based on carbon and phosphor atoms under the influence of an uniform magnetic field were investigated. The electronic spectrum shows an interesting behavior: the dependence of the energy levels on the magnetic field has a fractal structure. The calculation of the electronic spectrum was performed for common zigzag nanoribbons as well as their modifications with atomic vacancies and defects. Furthermore, the metallic properties of these modifications were verified. It was found that the metallic properties of the zigzag nanoribbons have a strong resistance against the structural perturbations.

# Covariant Quantum Spaces and gravity in the IKKT matrix model

Harold Steinacker

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I will discuss geometrical properties and physical perspectives of covariant quantum spaces, focusing on the fuzzy 4-sphere. These spaces realize noncommutative 4D geometries with local and global symmetries. The fluctuations on such a background within the IKKT matrix model lead to a higher spin theory, due to the underlying twisted bundle structure. Some aspects of the gravitational sector are discussed, leading to the linearized Einstein equations.



# Quantum properties of supersymmetric theories regularized by higher covariant derivatives

K.V. Stepanyantz

M.V.Lomonosov Moscow State University  
Faculty of Physics, Department of Theoretical Physics  
Leninskie Gory, Moscow, 119991, Russia

We investigate quantum corrections in  $\mathcal{N} = 1$  non-Abelian supersymmetric gauge theories, regularized by higher covariant derivatives. In particular, by the help of the Slavnov–Taylor identities we prove that the vertices with two ghost legs and one leg of the quantum gauge superfield are finite in all orders. This non-renormalization theorem is confirmed by an explicit one-loop calculation. By the help of this theorem we rewrite the exact NSVZ  $\beta$ -function in the form of the relation between the  $\beta$ -function and the anomalous dimensions of the matter superfields, of the quantum gauge superfield, and of the Faddeev–Popov ghosts. Such a relation has simple qualitative interpretation and allows suggesting a prescription producing the NSVZ scheme in all loops for the theories regularized by higher derivatives. This prescription is verified by the explicit three-loop calculation for the terms quartic in the Yukawa couplings.

# Quantum Double of Yangian of strange Lie superalgebra $Q_n$ and multiplicative formula for universal $R$ -matrix

Vladimir Stukopin

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Main result of report is proof of the multiplicative formula for universal  $R$ -matrix for Quantum Double of Yangian of strange Lie superalgebra  $Q_n$  type. We introduce the Quantum Double of the Yangian of the strange Lie superalgebra  $Q_n$  and define its PBW basis. We compute the Hopf pairing for the generators of the Yangian Double. From the formulas obtained for the Hopf pairing, we derive a structural factorized multiplicative formula for the universal  $R$ -matrix of the Yangian Double of the Lie superalgebra  $Q_n$ . After them we calculate coefficients in this multiplicative formula for universal  $R$ -matrix.

# From Heine to $q$ -Painlevé

Takao Suzuki

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The  $q$ -Painlevé VI equation ( $q$ - $P_{VI}$ ) was introduced by Jimbo and Sakai. It is known that  $q$ - $P_{VI}$  admits a particular solution in terms of the Heine's basic hypergeometric function  ${}_2\phi_1$ . In this talk, we propose a higher order generalization of  $q$ - $P_{VI}$  which admits a particular solution in terms of  ${}_{n+1}\phi_n$ . We also formulate  $\tau$ -functions for the generalized  $q$ - $P_{VI}$  on the root lattice  $Q(A_{2n+1})$  and show that they satisfy Hirota-Miwa type bilinear relations.

# The anatomy of coherent states *or* how a pure-pour mathematician see the topic

F.H. Szafraniec

I intend to distel and push forward the topic already generalised in:

A. Horzela and F.H. Szafraniec, A measure free approach to coherent states, *J. Phys. A: Math. Theor.* **45** (2012) 244018,

A. Horzela and F.H. Szafraniec, A measure free approach to coherent states refined, in Proceedings of the XXIX International Colloquium on Group-Theoretical Methods in Physics 2012, Tianjin, China, Nankai Series in Pure, Applied Mathematics and Theoretical Physics 11, 277.

# A factorization formula for rational mappings and tau functions

Tomoyuki Takenawa

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Joint work with: Adrian Stefan Carstea and Anton Dzhamay

This talk is the first of two talks describing the joint project with Adrian Stefan Carstea and Anton Dzhamay on geometric deautonomization. The goal of this project is to develop a systematic approach for deautonomizing discrete integrable mappings, such as the QRT mappings, to non-autonomous mappings in the discrete Painlevé family, based on the action of the mapping on the Picard lattice of the surface and a choice of a fiber in the elliptic fibration. In this talk we will explain the main ideas behind this approach and describe the technique that allows us to recover mapping in a factorized form from the known action on the divisor group. We also show this formula can be used for deriving bilinear equations on the tau functions.

# Construction of two-dimensional quantum field models through Longo-Witten endomorphisms

Yoh Tanimoto

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For a class of factorizing diagonal analytic S-matrix, we construct the full family of local observables satisfying the Haag-Kastler axioms, a mathematically satisfying framework of quantum field theory. These models are realized on the same Hilbert space as that of some copies of the free massive field, and can be regarded as deformations of the latter. We review various attempts to extend of this program, including some non-diagonal S-matrices and those with bound states.

# The Theory of Pseudodifferential Operators on the Noncommutative $n$ -Torus

Jim Tao

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The methods of spectral geometry are useful for investigating the metric aspects of noncommutative geometry and in these contexts require extensive use of pseudo-differential operators. In a foundational paper, Connes showed that, by direct analogy with the theory of pseudo-differential operators on  $\mathbb{R}^n$ , one may derive a similar pseudo-differential calculus on noncommutative  $n$  tori  $\mathbb{T}_\theta^n$ , and with the development of this calculus came many results concerning the local differential geometry of noncommutative tori for  $n = 2, 4$ , as shown in the groundbreaking paper in which the Gauss–Bonnet theorem on  $\mathbb{T}_\theta^2$  is proved and later papers. Certain details of the proofs in the original derivation of the calculus were omitted, such as the evaluation of oscillatory integrals, so we make it the objective of this paper to fill in all the details. After reproving in more detail the formula for the symbol of the adjoint of a pseudo-differential operator and the formula for the symbol of a product of two pseudo-differential operators, we define the corresponding analog of Sobolev spaces for which we prove the Sobolev and Rellich lemmas. We then extend these results to finitely generated projective right modules over the noncommutative  $n$  torus.

# On curvature squared invariants in 6D $\mathcal{N} = (1, 0)$ supergravity

Gabriele Tartaglino-Mazzucchelli

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Celestijnenlaan 200D, B-3001 Leuven, Belgium

We review the construction of the supersymmetric completion of all the curvature squared invariants in 6D  $\mathcal{N} = (1, 0)$  supergravity, including a novel Weyl<sup>2</sup> term. The building blocks in our analysis are superspace and superconformal tensor calculus techniques. We will then comment on applications of these invariants.



# Algebraic Approach to (Super)Conformal Quantum Mechanics

Francesco Toppan

Centro Brasileiro de Pesquisas Físicas - CBPF  
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Two equivalent approaches for quantizing (super)conformal quantum mechanical systems with Calogero potentials (and possibly an extra damping DFF oscillatorial term) are discussed. They produce simple Lie superalgebras as spectrum-generating algebras. The first approach is based on the canonical quantization of world-line superconformal invariant actions. The second approach is based on the symmetry (super)algebra of Partial Differential Equations.

Examples with spectrum-generating superalgebras  $sl(2|1)$  and  $D(2, 1; \alpha)$  are discussed.

# Irreducible representations of finite dimensional quotient algebras of the braid group $B_3$ .

Anastasiia Trofimova

NRU HSE, Moscow,  
Usacheva, 6

I will classify irreducible representations of the three string braid group for which the generators of elementary braids satisfy 5-th order minimal polynomial with the pairwise different eigenvalues. These representations are described explicitly in the diagonal basis for the Jucys-Murphy elements. The construction allows to obtain the semisimplicity criteria for the corresponding finite dimensional quotient algebras of  $C[B_3]$ .

# Determinant structure for $\tau$ -functions of holonomic deformation of linear differential equations

Teruhisa Tsuda

Department of Economics, Hitotsubashi University,  
Tokyo, Japan

We present a determinant formula for the ratio of tau-functions of a holonomic deformation of linear differential equations by using Hermite–Padé approximations. This talk is based on a joint work with Masao Ishikawa and Toshiyuki Mano.

# On theta function solutions for nonlocal soliton equations

Yohei Tutiya

Kanagawa Institute of Technology  
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We will see that ILW equation and Periodic ILW equation with discrete Laplacian possess theta function solutions whose genera are greater than two.

# On Quantum Cosmology in Teleparallel Gravity

Sérgio Costa Ulhoa

Universidade de Brasília  
Campus Darcy Ribeiro, Distrito Federal  
Brazil

Quantum cosmology has been an important field of research for the last thirty years. It is based on the Wheeler-de Witt equation, which introduces the notion of operator to the scale factor of the Universe. On the other hand it has some flaws, for instance we name a few, such as the problem of time and energy. In order to get over some difficulties we use Teleparallel Gravity to construct a well defined expression for gravitational energy and apply the Weyl quantization procedure to obtain an equation for the quantum dynamics of the Universe. Such a procedure was constructed as a non-commutativity of spatial coordinates. We show that the non-commutativity in time leads to problems to establish the quantum dynamics.

# Pluri-Lagrangian systems

Mats Vermeeren

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Given the prevalence of Hamiltonian structures in integrable systems theory, Lagrangian structures play a surprisingly small role in the existing literature. In the past decade, this has begun to change with the introduction of *Lagrangian multiform* or *pluri-Lagrangian* systems. These are either hierarchies of commuting differential equations or multidimensionally consistent lattice systems, described by a variational principle where not just the fields are varied but also the geometry of the action integral. Very recently some steps were made towards a quantum version of the theory, but this talk will focus on the classical context, as its aim is to give an overview of the central ideas of the pluri-Lagrangian formalism.

# On the invariants of birational maps

C.M. Viallet

Centre National de la Recherche Scientifique & UPMC Sorbonne Universités  
Paris, France

Joint work with: N. Joshi

Abstract: We are used to autonomous discrete integrable systems having rational invariants, with the paradigmatic example of the so called QRT maps. I will present an example of integrable birational map, having both rational and non-rational invariants. We get a characterisation of integrability as well as an information on the nature of the invariants by a degree analysis: the vanishing of the entropy shows integrability, and the non quadratic growth of the sequence of degrees signals that it is of a special kind.

# Integrable field theories and dihedral affine Gaudin models

Benoit Vicedo

School of Physics, Astronomy and Mathematics,  
University of Hertfordshire, College Lane, Hatfield AL10 9AB, UK

I will explain how a very broad class of (non-ultralocal) classical integrable field theories can be reformulated as classical Gaudin models associated with an affine Kac-Moody algebra and with dihedral symmetry. This provides a natural setting within which to address the problem of quantisation of these theories and may also furnish a general framework for understanding the massive ODE/IM correspondence.



# The quantum rotation number for 2D integrable systems.

San Vu Ngoc

Joint work with: M. Dauge and M. Hall

The rotation number of an integrable Hamiltonian system is a well-known dynamical quantity, whose irrationality can ensure the persistence of integrable dynamics under perturbation (KAM theory). Motivated by recent results on spectral asymptotics of some nonselfadjoint operators by Hitrik-Sjstrand, we introduce a purely spectral number, the quantum rotation number, associated to a pair of commuting operators. We prove that this number recovers the classical rotation number in the semiclassical limit, and investigate the case of semitoric systems, including numerics.

# Kaluza-Klein reduction of Supergravity: Geometric approach

Jan Vysoky

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Czech Republic

An idea to obtain the gauge theory of electromagnetism in the presence of a gravitational field from the simpler theory in higher dimensions dates back to 1919. Various generalizations of this concept allow one to obtain a non-Abelian Yang-Mills model in this way. In particular, we seek for such procedure for so called heterotic supergravity. It turns out that equations of motion of such theory can be obtained as geometrical constraints imposed onto a generalized Ricci tensor of certain Levi-Civita connection on Courant algebroids. This allows one to use a known procedure of Courant algebroid reductions to find the suitable reduction of supergravity theories.

# Exact solution of the relativistic quantum Toda chain

Yupeng Wang

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The relativistic quantum Toda chain model is studied with the generalized algebraic Bethe Ansatz method. By employing a set of local gauge transformations, proper local vacuum states can be obtained for this model. The exact spectrum and eigenstates of the model are thus constructed simultaneously.

# From singularity patterns to algebraic entropies

Ralph Willox

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Joint work with: Takafumi Mase, Alfred Ramani and Basil Grammaticos

In the first part of this talk I will describe how, for confining three point mappings, the singularity patterns can be used to calculate the algebraic entropy of a mapping in a rigorous way, using only simple arithmetic. I will explain the algebro-geometric underpinnings of this so-called ‘express method’ and, in the second part of the talk, I will describe how this method can be extended to certain non-confining three point mappings.

# Higher order superintegrable systems: a new Painlevé conjecture

Pavel Winternitz

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The *Painlevé conjecture* has played an important role in the theory of infinite dimensional integrable systems (soliton theory) where it has been highly successful in identifying equations solvable by the inverse scattering transform. Here we present and justify a different Painlevé conjecture in the theory of finite-dimensional quantum superintegrable systems. We present a review of some recent results on "exotic quantum potentials" in two-dimensional Euclidean space that allow the separation of variables in the Schrödinger equation and allow an additional integral of motion of order  $N > 2$ . We call them exotic because they do not satisfy any linear PDE. Instead, they satisfy nonlinear ODEs. For  $N = 3, 4$  and  $5$  it turns out that these nonlinear ODEs always have the Painlevé property. We conjecture that this is true for all values of  $N$ .

# Capelli elements of the group algebra

Naoya Yamaguchi

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Japan

Inspired by the Capelli identities for group determinants obtained by Tôru Umeda, we give a basis of the center of the group algebra of any finite group by using Capelli identities for irreducible representations. The Capelli identities for irreducible representations are modifications of the Capelli identity. These identities lead to Capelli elements of the group algebra. These elements construct a basis of the center of the group algebra.

# Darboux transformation and soliton solutions for the (2+1)-dimensional two-component nonlinear Schrödinger equations

K.R. Yesmakhanova, G.T. Bekova, G.N. Shaikhova and R. Myrzakulov

Eurasian International Center for Theoretical Physics and Department of General & Theoretical Physics  
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In this paper, we introduce a new Lax representation for the (2+1)-dimensional two-component nonlinear Schrödinger equations (TNLSE). Darboux transformation (DT) of this system is constructed. We derive soliton solutions from trivial "seed" solutions by DT.

# Cyclotomic Gaudin models and hyperplane arrangements

Charles Young

University of Hertfordshire, UK

Given an automorphism of a simple Lie algebra  $\mathfrak{g}$ , I will define a cyclotomic Gaudin algebra. It is a large commutative algebra, generated by a hierarchy of cyclotomic Gaudin Hamiltonians which reduce to the usual Gaudin Hamiltonians when the automorphism is the identity.

There are known to be deep connections between Gaudin models and hyperplane arrangements. I will briefly recall these links. Then I will describe a natural yet non-trivial generalization, to the cyclotomic setting, of a key result of Schechtman and Varchenko relating the flag and Aomoto complexes of discriminantal arrangements to chain complexes for Kac-Moody Lie algebras.

This talk is based on joint work B. Vicedo (arxiv:1409.6937 and arxiv:1611.09059) and with A. Varchenko (arXiv:1603.07125).



# Baxter Q operators and asymptotic representations of quantum groups

Huafeng Zhang

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Joint work with: Giovanni Felder

We construct the *asymptotic representations* for Felder's elliptic quantum groups, and use them to define the Baxter Q-operators for the associated face-type quantum integrable systems as transfer matrices. These operators satisfy three-term TQ relations, from which we deduce the Bethe Ansatz Equations under genericity conditions.

# Commutators of projectors and projective geometry

Ilya Zhdanovskiy

MIPT

Dolgoprudniy, Moscow Region, Russia

Consider algebra generated by two sets of orthogonal idempotents, satisfying to some commutator's relations. I will talk about representations of this algebra, its quantum-mechanical, algebraic and geometric properties. This talk is based on joint work with Anna Kocherova.

# A Theory of Gravity and General Relativity based on Quantum Electromagnetism

J.X. Zheng-Johansson

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We predict the presence of a universal attractive depolarisation radiation (DR) Lorentz force between two quantum entities, each being either a light quantum or IED matter particle, in a vacuonic dielectric vacuum based on first principles solutions in a unified framework of electromagnetism and quantum mechanics. Given two quantum entities  $i = 1, 2$  of either kind of characteristic frequencies  $\nu_i^0$ , masses  $m_i^0 = h\nu_i^0/c^2$  and separated at a distance  $r^0$  being (infinitely) large, the DR Lorentz force obeys an inverse square law  $F = -\mathcal{G}m_1^0m_2^0/(r^0)^2$ , where  $\mathcal{G} = \chi_0^2e^4/12\pi^2\epsilon_0^2\rho_\lambda$ ,  $\chi_0$  is the susceptibility and  $\rho_\lambda$  the reduced linear mass density of the dielectric vacuum against an empty space. This force  $F$  resembles in all respects Newton's gravity, and hence  $\mathcal{G}$  the gravitational constant  $G$ . The DR wave fields and hence the gravity is propagated in the dielectric vacuum at the speed of light  $c$ ; these can not be shielded by matter. Given a massive object of mass  $M$ , a test object of mass  $m^0$  at  $r^0$  apart is therefore gravitated by all its building IED particles directly, by a total gravitational potential  $V = -\partial F/\partial r^0 = -GMm^0/r^0$ . For a non-trivial finite  $V$ , the eigen Hamiltonian solution combined with the wave nature of the gravity yield a set of general relativistic equations including time (dilation)  $t = t^0/(1 - GM/r^0c^2)$ , distance (elongation), and modified Newton's law of gravity and Einstein's mass energy relation. Applications of these give predictions of the general relativistic effects manifested in the three or so classical-test experiments of general relativity (GR), which are in direct agreement with the experiments and the predictions based on Einstein's GR. This research has been motivated by an open remark by Professor G 't Hooft regarding the completeness of a gravity theory.

# Schubert calculus and the bootstrap equation, I

Paul Zinn-Justin

Melbourne University  
Australia

Joint work with: Allen Knutson

We give background on “Schubert calculus”, the Littlewood-Richardson structure constants of the cohomology of the Grassmannian, and many of its generalizations:  $T$ -equivariant, to  $K$ -theory, quantum cohomology, other flag manifolds. Many of these have been given formulæ in terms of the “puzzles” introduced in [Knutson-Tao '03].

In particular, the equivariant extension can be interpreted as a degenerate integrable model with parameters, based on the standard action of  $\mathfrak{sl}_3$ . The corresponding nondegenerate model includes  $K$ -theory and further generalizations.

# Classical-quantum dualities in integrable systems

Andrei Zotov

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We review quantum-classical dualities between classical integrable many-body systems and quantum spin chains. Quantum-quantum and classical-classical counterparts of this phenomenon are discussed as well as possible applications.

# Modular form identities from vertex operator algebras

Alexander Zuevsky

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We will give a short reminder for vertex operator algebra notion and corresponding characters. Then we discuss algebraic methods for explicit computation of the partition and correlation functions. Finally, a general way to find number theory identities for related modular forms will be given.

# Spectra, automata and discrete analogues of the KdV equation

Andrzej Zuk

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Joint work with: T. Kato, S. Tsujimoto

Box-Ball systems are discrete analogues of the KdV equation. We prove that their evolution can be described by automata. With these automata we associate self-adjoint operators. We relate spectral properties of these operators with L2 Betti numbers of closed manifolds.

# Mock-Lie algebras

Pasha Zusmanovich

University of Ostrava, Ostrava, Czech Republic

Mock-Lie algebras are commutative algebras satisfying the Jacobi identity. These strange cousins of Lie algebras turn out to be related to several interesting questions: Yang-Baxter equation, new realizations of certain physically-motivated Lie algebras, etc.