Superintegrable systems on Poisson manifolds

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One usually considers superintegrable (or non-commutative completely integrable) systems on symplectic manifolds. However, their definition implies rather restrictive condition $n = k + m$ where n is a dimension of a symplectic manifold, k is a dimension of a pointwise Lie algebra of a superintegrable system and m is its corank. To overcome this difficulty, we aim to consider superintegrable systems on Poisson manifolds. A key point is that invariant submanifolds of a superintegrable system are maximal integral manifolds of a certain partially integrable system. Therefore, we follow our analysis of partially integrable systems on Poisson manifolds [1,3]. Let Z be an n-dimensional smooth manifold. Let us consider a k-dimensional real Lie algebra A of complete vector fields on Z which are linearly independent almost everywhere. It is called the dynamical algebra. We show that, under certain conditions, there exists a saturated neighborhood U of an invariant submanifold M of a dynamical algebra A which can be provided with a Poisson structure W of rank r so that elements of A restricted to U are Hamiltonian vector fields of some function on U. These functions constitute a partially superintegrable system (A, W) on U so that the relation $k + m = r \leq n$ holds. Certainly, such a Poisson structure need not be unique. A family of these Poisson structures is described. If $r = n$, this is the case of superintegrable systems on a symplectic manifold. If $r < n$, one can show that a partially superintegrable system on a Poisson manifold (Z, W) yields superintegrable systems on leaves of the corresponding symplectic foliation of Z. The generalized MishchenkoFomenko theorem on action-angle coordinates in a case of symplectic superintegrable systems [2,3] is extended to partially superintegrable systems on Poisson manifolds. An example of a partially superintegrable system on Poisson manifolds that we analyze in detail is a superintegrable system with parameters whose phase space is a fiber bundle $Z \to \Sigma$ over a manifold Σ of parameters.

References. [1] G. Giachetta, L. Mangiarotti and G. Sardanashvily, Bi-Hamiltonian partially integrable systems, J. Math. Phys. 44 (2003) 1984. [2] E. Fiorani and G. Sardanashvily, Noncommutative integrability on noncompact invariant manifolds, J. Phys. A 39 (2006) 14035. [3] G. Sardanashvily, Handbook of Integrable Hamiltonian Systems, Editorial URSS, Moscow, 2015.