

# Solutions of the Schrodinger equation in terms of the Heun functions

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We review the cases for which the Schrödinger equation is solved in terms of the general and confluent Heun functions. We present the possible choices for the coordinate transformation that provide energy-independent potentials that are proportional to an energy-independent continuous parameter and have a shape independent of that parameter. In contrast to the hypergeometric case, no Heun potential can in general be transformed into another one by specifications of the involved parameters [1]. There are several independent potentials that present distinct generalizations of either a hypergeometric or a confluent hypergeometric classical potential, some potentials possess sub-cases of both hypergeometric types, and others possess particular conditionally integrable ordinary or confluent hypergeometric sub-potentials. We present several examples of explicit solutions for the latter potentials. We show that there exist other exactly or conditionally integrable sub-potentials the solution for which is written in terms of simpler special functions. However, these are solutions of different structure. For instance, there are sub-potentials for which each of the two fundamental solutions of the Schrödinger equation is written in terms of irreducible combinations of hypergeometric functions [2-4]. Several such potentials are derived with the use of extended Heun equations [5]. A complementary approach is the termination of the hypergeometric series expansions of the solutions of the Heun equations (e.g., [6]).

References 1. A.M. Ishkhanyan, "Schrödinger potentials solvable in terms of the confluent Heun functions", *Theor. Math. Phys.* (2016), in press. 2. A.M. Ishkhanyan, "Exact solution of the Schrödinger equation for the inverse square root potential", *Eur. Phys. Lett.* 112, 10006 (2015). 3. A.M. Ishkhanyan, "The Lambert-W step-potential - an exactly solvable confluent hypergeometric potential", *Phys. Lett. A* 380, 640-644 (2016). 4. A.M. Ishkhanyan, "A conditionally exactly solvable generalization of the inverse square root potential", arXiv:1511.03565 (2016). 5. A.Ya. Kazakov and S.Yu. Slavyanov, "Euler integral symmetries for the confluent Heun equation and symmetries of the Painlevé equation PV", *Theor. Math. Phys.* 179, 543 (2014). 6. A.M. Ishkhanyan, "The third exactly solvable hypergeometric quantum-mechanical potential", arXiv:1602.07685 [quant-ph] (2016).