On a Microscopic Representation of Space-Time V

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In previous parts of this publication series, starting from the Dirac algebra and SU$*$(4), the 'dual' compact rank-3 group SU(4) and Lie theory, we have developed some arguments and the reasoning to use (real) projective and (line) Complex geometry directly. Here, we want to extend this approach further in terms of line and Complex geometry and give some analytical examples. As such, we start from quadratic Complexe which we've identified in part IV already as yielding naturally the 'light cone' $x_1^2 + x_2^2 + x_3^2 - x_0^2 = 0$ while being related to (homogeneous) point coordinate descriptions x_α^2 and to infinitesimal dynamics by tetrahedral Complexe (or line elements) which thus naturally introduce projective transformations by preserving anharmonic ratios. We summarize some old work of ? relating quadratic Complexe to optics and discuss briefly their relation to spherical (and Schrodinger-type) equations as well as an obvious interpretation based on homogeneous coordinates and relations to conics and second order surfaces. Discussing (linear) symplectic symmetry and line coordinates, the main purpose and thread within this paper, however, is the identification and discussion of special relativity as direct invariance properties of line/Complex coordinates as well as their relation to 'quantum field theory' by complexification of point coordinates or Complexe. This can be established by the Lie mapping which relates lines/Complexe to sphere geometry so that SU(2), SU(2)$\times$U(1), SU(2)$\times$SU(2) and the Dirac spinor description emerge without additional assumptions. We give a short outlook in that quadratic Complexe are related to dynamics e.g. power expressions in terms of six-vector products of Complexe, and action principles may be applied. (Quadratic) products like $F^{\mu\nu}F_{\mu\nu}$ or $F^{a\,\mu\nu}F^a_{\mu\nu}$, $1 \leq a \leq 3$ are natural quadratic Complex expressions which may be extended by line constraints $\lambda k \cdot \epsilon = 0$ with respect to an 'action principle' so that we identify 'quantum field theory' with projective or line/Complex geometry having applied the Lie mapping. We close with a brief outlook on related mathematics, physics and 'higher geometry'.