

# Generalization of the pseudo-diffusion equation of twisting, yielding realization of $\mathfrak{so}(n, m)$ and two contractions

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## Abstract

We study the symmetry algebra  $\mathfrak{g}$  of the following  $(n+m)$ -dimensional generalized pseudo-diffusion equation:

$$L u \equiv \left[ \frac{\partial}{\partial t} - \frac{1}{4} \sum_{k=1}^n \frac{\partial^2}{\partial x_k^2} - \frac{\gamma}{4t^2} \sum_{r=1}^m \frac{\partial^2}{\partial p_r^2} \right] u(x_k, p_r, t) = 0.$$

For  $n = m = 1$  and  $\gamma = 1$  this equation describes the dependence of the  $Q(x, p, t)$ -function on  $t = e^{2y}$ , where  $y$  is the squeezing parameter. This will lead me to give a review of coherent states and of squeezing operators, and their relation to uncertainty relations.

A sub-algebra of  $\mathfrak{g}$  for  $\gamma > 0$  yields a *realization of  $\mathfrak{so}(n, m)$  which depends explicitly on the coupling constant  $\gamma$* . By taking the limit  $\gamma \rightarrow 0$  we obtain an ‘internal’ or ‘natural’ contraction of  $\mathfrak{so}(n, m)$  to  $\mathfrak{iso}(n, m)$ , similar to the contraction of the dynamical symmetry algebra of the hydrogen atom. We also describe ‘external’ contractions of  $\mathfrak{so}(n, m)$ , such as

$$\mathfrak{so}(n, m) \rightarrow \mathfrak{so}(n, m - 4) \oplus (\mathfrak{h}_{n+m-4} \oplus \mathfrak{h}_{n+m-2}),$$

where  $\mathfrak{h}_n$  is the  $(2n + 1)$ -dimensional Heisenberg-Weyl algebra.