Yang-Baxter relations with orthogonal or symplectic symmetry

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Abstract. Solutions of the Yang-Baxter relations with orthogonal or symplectic symmetry are studied emphasizing the analogies of both cases. Starting from Jordan-Schwinger representations the distinguishing features of the spinor representation and its symplectic analogon are shown. The corresponding L matrix and the spinorial R operator are discussed.

1. Introduction

The R matrix obeying the Yang-Baxter (YB) relation and representing the orthogonal or symplectic algebras in their fundamental representation is not linear in the spectral parameter [1, 2, 4]. The RLL case of the YB relation involving this fundamental R matrix together with the L matrix operators of the form $L_{a,b}(u) = u\delta_{ab} + M_{ab}$ does not hold for an arbitrary representation with the generators M_{ab}. The spinor representation of the orthogonal algebra with $M_{ab} = \frac{1}{4}[\gamma_a, \gamma_b]$ is a distinguished case, where the RLL relation is obeyed. Also the spinorial R matrix, intertwining two spinor representations, is known [3]. This and other representations of the orthogonal algebra distinguished in this sense and the corresponding R operators have been considered recently and the spinorial R operator has been analysed in detail [6].

We consider the YB relations with symplectic symmetry relying on the known analogies to the orthogonal case. We use the Jordan-Schwinger (JS) type of representations, where the generators M_{ab} are composed of canonical pairs x_a, ∂_a obeying the Heisenberg commutation relation in the usual bosonic or in the fermionic/Grassmann version [5].

2. Jordan-Schwinger representations

From $2n(+1)$ canonical pairs $x_a, \partial_a, a = 1, ..., 2n(+1)$ one can construct operators obeying the $g\ell(2n(+1))$ algebra relations. We consider two versions,

$$E_{ab}^+ = x_a \partial_b, \ E_{ab}^- = -\partial_a x_b, \tag{1}$$

related by the elementary canonical transformations

$$\mathcal{C} E_{ab}^+ \mathcal{C}^{-1} = E_{ab}^- \tag{2}$$

defined as

$$\mathcal{C}_a \begin{pmatrix} x_a \\ \partial_a \end{pmatrix} \mathcal{C}_a^{-1} = \begin{pmatrix} \partial_a \\ -x_a \end{pmatrix}, \quad \mathcal{C} = \prod \mathcal{C}_a. \tag{3}$$

We consider the action of the constructed generators on functions of x_a. On the linear space spanned by x_a we have the fundamental representations. The two representations are dual to each other in the following sense. If the first set of generators acts on $\mathcal{V}^+$ as infinitesimal transformations and the second one on $\mathcal{V}^-$, both spanned by the x_a as basis vectors, the scalar product of $\mathbf{x}_1 \in \mathcal{V}^+$ and $\mathbf{x}_2 \in \mathcal{V}^-$ defined as $(\mathbf{x}_1\mathbf{x}_2) = \sum_a x_{1,a} x_{2,a}$ is invariant in the sense

$$[E_{ab}^{(1)+} + E_{ab}^{(2)-}, (\mathbf{x}_1\mathbf{x}_2)] = 0. \tag{4}$$

In both cases the algebra commutes with $\sum N_a$, $N_a = x_a \partial_a$.

We need also a fermionic/Grassmann version of the JS representation based on odd canonical pairs $\theta_a, \partial_a^\theta$, $[\theta_a, \partial_b^\theta]_+ = \delta_{ab}$. The relations of $g\ell(2n(+1))$ are fulfilled as well by

$$\mathrm{E}_{ab}^+ = \theta_a \partial_b^\theta, \quad \mathrm{E}_{ab}^- = \partial_a^\theta \theta_b$$

The elementary canonical transformations are now

$$\mathcal{C}_a^{(\alpha)} \begin{pmatrix} \theta_a \\ \partial_a^\theta \end{pmatrix} \mathcal{C}_a^{(\alpha)} = \begin{pmatrix} \alpha \partial_a^\theta \\ \alpha^{-1} \theta_a \end{pmatrix}.$$

We shall use the cases $\alpha = \pm 1$. The bilinear form is the same one with the bosonic coordinates substituted by the fermionic ones.

Now we consider the case that both sets $E_{ab}^\pm$ act on the same space. The sum is invariant with respect to the elementary canonical transformation,

$$\mathcal{C} F_{ab} \mathcal{C}^{-1} = F_{ab}, \quad F_{ab} = E_{ab}^+ + E_{ab}^- + \delta_{ab} = x_a \partial_b - x_b \partial_a,$$

and the scalar product of two elements of this space is invariant,

$$[F_{ab}, (\mathbf{x}_1\mathbf{x}_2)] = 0.$$

In this way in the JS case the subalgebra $so(2n(+1))$ of $g\ell(2n(+1))$ can be considered as the one invariant with respect to the elementary canonical transformation.

In order to have a smooth comparison with the symplectic case we rewrite the bilinear form as a result of relabelling the indices $1, 2, ...2n + (1) \to -n, -n + 1, ..., (0), ..., n - 1, n$. There is a difference between the even and odd cases. In the latter the index value 0 is included and in the even case it is excluded. The bilinear form reads now

$$(\mathbf{x}_1\mathbf{x}_2) = \sum_{-n}^{n} x_{1,a} x_{2,-a}, \tag{5}$$

and the generators of the $so(2n(+1))$ subalgebra are

$$F_{ab} = E_{ab}^+ + E_{-a,-b}^-. \tag{6}$$

They are invariant with respect to the elementary canonical transformation including the index value reflection.

$$\mathcal{C} = \prod \mathcal{C}_a,$$

where in the bosonic case we have

$$\mathcal{C}_a \begin{pmatrix} x_a \\ \partial_a \end{pmatrix} \mathcal{C}_a^{-1} = \begin{pmatrix} \partial_{-a} \\ -x_{-a} \end{pmatrix}, \tag{7}$$

and the fermionic case

$$\mathcal{C}_a^{(\pm)} \begin{pmatrix} \theta_a \\ \partial_a^\theta \end{pmatrix} \mathcal{C}_a^{(\pm)} = \begin{pmatrix} \pm\partial_{-a}^\theta \\ \pm\theta_{-a} \end{pmatrix}.$$

The subalgebra $sp(n)$ of $g\ell(2n)$ is defined as the one commuting with the bilinear form

$$< \mathbf{x}_1\mathbf{x}_2 > = \sum_{-n}^{n} \varepsilon_a x_{1,a} x_{2,-a}$$

$$[G_{ij}, < \mathbf{x}_1\mathbf{x}_2 >] = 0, \ G_{ij} = E_{ab}^+ + \varepsilon_a\varepsilon_b E_{-a,-b}^-$$

where ε_a denotes the sign of the index a. We have an analogous statement about the characterisation of $sp(n)$ as the subalgebra of $g\ell(2n)$, but the elementary canonical transformations enter with an index sign dependent modification. In the bosonic case we have

$$\tilde{\mathcal{C}} = \prod_1^n \mathcal{C}_a \prod_{-n}^{-1} \mathcal{C}_a^{-1}, \tag{8}$$

and in the fermionic case

$$\tilde{\mathcal{C}} = \prod_1^n \mathcal{C}_a^+ \prod_{-n}^{-1} \mathcal{C}_a^-. \tag{9}$$

Thus the subalgebra $sp(n)$ for JS representations can be regarded as the one invariant with respect to this canonical transformation $\tilde{\mathcal{C}}$.

It is important to notice that there are linear combinations of the underlying Heisenberg generators, invarant (up to sign) under the above elementary canonical transformations.

$$\mathcal{C}(\partial_a \pm \theta_{-a})\mathcal{C} = \pm(\partial_a \pm \theta_{-a}), \tag{10}$$

$$\tilde{\mathcal{C}}(\partial_a \pm \varepsilon_a x_{-a})\tilde{\mathcal{C}}^{-1} = \mp(\partial_a \pm \varepsilon_a x_{-a}).$$

The first relation involving $\mathcal{C}$ is relevant for orthogonal and the second with the index sign dependent $\tilde{\mathcal{C}}$ in the symplectic case. Interchanging the bosonic with the fermionic versions we do not obtain invariance, but the two linear combinations transform into each other:

$$\mathcal{C}(\partial_a \pm x_{-a})\mathcal{C}^{-1} = \pm(\partial_a \mp x_{-a}), \quad \tilde{\mathcal{C}}(\partial_a \pm \varepsilon_a\theta_{-a})\tilde{\mathcal{C}} = \mp(\partial_a \mp \varepsilon_a\theta_{-a}).$$

3. Linear transformations separating generators

We shall study linear transformations of the canonical pairs x_a, ∂_a or $\theta_a, \partial_a^\theta$. Let us write the bosonic case

$$x_a' = A_{ab}x_b + B_{ab}\partial_b, \ \partial_a' = C_{ab}x_b + D_{ab}\partial_b.$$

We may calculate the commutation relations of the new combinations in terms of the transformation matrix blocks A, B, C, D.

$$[x_a', \partial_b'] = (BC^T - AD^T)_{ab}, \ \ [x_a', x_b'] = (BA^T - AB^T)_{ab}, \ \ [\partial_a', \partial_b] = (DC^T - CD^T)_{ab}.$$

Now we have the important case of linear canonical transformations, where the latter two (anti) commutators vanish and the first one is proportional to the unit matrix.

We shall consider the case of the transformation, where the opposite occurs. Namely, the first commutator vanishes and the other two are proportional to the distinguished matrix defining the invariant scalar product. In the orthogonal case it is the unit matrix I or the matrix $\hat{\varepsilon}^+$ with

the elements $\varepsilon_{ab}^+ = \delta_{a,-b}$ if one prefers to label the components by $-n, ..., +n$. In the symplectic case it is the antisymmetric matrix $\hat{\varepsilon}^-$, $\varepsilon_{ab}^- = \varepsilon_a \delta_{a,-b}$.

In the orthogonal case this can work only in the fermionic version, since the distinguished matrix is symmetric. In the symplectic case it can work only in the bosonic version since the distinguished matrix is anti-symmetric. In the following we specify the transformations as $A = I, B = -\varepsilon, C = -\varepsilon, D = I$. Then we have for the symplectic case

$$[x'_a, \partial'_b] = 0, \ [x'_a, x'_b] = -2\varepsilon_a \delta_{a,-b}, \ [\partial'_a, \partial'_b] = 2\varepsilon_a \delta_{a,-b},$$

and for the orthogonal case

$$[\theta'_a, \partial_b^{\theta,'}]_+ = 0, \ [\theta'_a, \theta'_b]_+ = 2\delta_{a,-b}, \ [\partial'_a, \partial'_b]_+ = -2\delta_{a,-b}.$$

Now let us rewrite the JS generators in the transformed canonical variables. In the orthogonal case we have

$$M_{ab} = \theta_a \partial_b^\theta - \theta_b \partial_a^\theta = M_{\gamma \ ab} - \tilde{M}_{\gamma \ ab}, \tag{11}$$

with the notations

$$\gamma_a = \theta'_a = \theta_a + \partial_a^\theta, \ \ \tilde{\gamma}_a = -\partial_a^{\theta'} = \theta_a - \partial_a^\theta. \tag{12}$$

$$M_{\gamma \ ab} = \frac{1}{4}[\gamma_a, \gamma_b], \ \ \ \tilde{M}_{\gamma \ ab} = \frac{1}{4}[\tilde{\gamma}_a, \tilde{\gamma}_b]. \tag{13}$$

In particular we have

$$[M_{\gamma \ ab}, \tilde{M}_{\gamma \ cd}] = 0$$

Therefore the two terms in (11) obey the orthogonal algebra relations independently. γ_a may be identified with the conventional gamma matrices. Indeed, they generate the Clifford algebra in $2n(+1)$ dimensions.

Note that the two sets of Clifford generators, γ_a and $\tilde{\gamma}_a$ are just the linear combinations (10) of the fermionic Heisenberg generators that are invariant up to sign with respect to the fermionic elementary canonical transformations (7).

In the symplectic case we have

$$G_{ab} = x_a \partial_b - \varepsilon_a \varepsilon_b x_{-b} \partial_{-a} = G_{\Gamma \ ab} + \tilde{G}_{\Gamma \ ab}, \tag{14}$$

with the notations

$$\tilde{\Gamma}_a = x'_a = x_a + \varepsilon_a \partial_{-a}, \ \Gamma_a = \partial'_a = \partial_a - \varepsilon_a x_{-a}, \tag{15}$$

$$\tilde{G}_{\Gamma \ ab} = \frac{1}{4}\varepsilon_b[\tilde{\Gamma}_a, \tilde{\Gamma}_{-b}]_+, \ \ G_{\Gamma \ ab} = \frac{1}{4}\varepsilon_a[\Gamma_{-a}, \Gamma_b]_+. \tag{16}$$

In particular we have in analogy

$$[G_{\Gamma \ ab}, \tilde{G}_{\Gamma \ cd}] = 0. \tag{17}$$

Therefore the two terms in (14) obey the symplectic algebra relations independently. Γ_a generate the symplectic analogon of the Clifford algebra,

$$[\Gamma_a, \Gamma_b] = [\partial'_a, \partial'_b] = 2\varepsilon_a \delta_{a,-b} = 2\epsilon_{ab}. \tag{18}$$

We recall that the two sets of symplectic Clifford generators Γ_a and $\tilde{\Gamma}_a$ are just the linear combinations of the bosonic Heisenberg generators (10) invariant with respect to the sign dependent bosonic elementary canonical transformation (8).

In this way we have seen that the JS generators of $sp(n), so(2n)$ both separate in two terms, commuting and generating the equivalent algebra independently. Each set of such separated generators is built from $2n$ quasi-Clifford generators $\gamma_a, a = -n, ..., +n$.

The latter can also be viewed as n Heisenberg pairs, $\gamma_{-a}, \gamma_a, a = 1, ..., n$, bosonic in the symplectic case and fermionic in the orthogonal case. And these quasi-Clifford generators can be used for building JS representations of $g\ell(n)$.

4. Fundamental YB matrices and L operators

In both orthogonal and symplectic cases there are analoga of Yang's fundamental Yang-Baxter matrix $R_{12} = uI + P_{12}$ obeying the Yang-Baxter relation in the form

$$R_{b_1 b_2}^{a_1 a_2}(u) R_{c_1 b_3}^{b_1 a_3}(u+v) R_{c_2 c_3}^{b_2 b_3}(v) = R_{b_2 b_3}^{a_2 a_3}(v) R_{b_1 c_3}^{a_1 b_3}(u+v) R_{c_1 c_2}^{b_1 b_2}(u). \tag{19}$$

In the orthogonal and symplectic fundamental R matrices the existence of the invariant tensor causes a third term in the corresponding expressions, and related to this the dependence on the spectral parameter involves its second power. The well known results [1, 2, 4] are easily checked starting from the ansatz

$$R_{b_1 b_2}^{a_1 a_2}(u) = u(u+\beta)\delta_{b_1}^{a_1}\delta_{b_2}^{a_2} + (u+\beta)\delta_{b_2}^{a_1}\delta_{b_1}^{a_2} - u\varepsilon_{a_1}\varepsilon_{b_2}\delta^{a_1,-a_2}\delta^{b_1,-b_2}, \tag{20}$$

where we have denoted $\varepsilon_a = 1$ in the orthogonal case and $\varepsilon_a = sign(a)$ in the symplectic case. One finds that the above YB relation holds if

$$\beta = \beta_\pm = n \mp 1 \tag{21}$$

for the orthogonal or symplectic case, respectively.

Above we have reformulated the orthogonal case to get closer to the symplectic one by choosing the index range $a, b = -n, ...(0)..., n$ and have written the scalar product with the metric $\delta_{a,-b}$. The anti-symmetry relation then reads

$$M_{ab} = -M_{-b-a}, \ a, b = -n, ...(0)..., n \tag{22}$$

If we agree to denote by ε_a in the symplectic case $sign(a)$ and just 1 in the orthogonal case then the anti-symmetry relation in both cases reads

$$G_{ab} = -\varepsilon_a \varepsilon_b G_{-b,-a}, \tag{23}$$

and the invariant tensor defining the scalar product can be written as $\varepsilon_{a,b} = \varepsilon_a \delta_{a,-b}$. It is symmetric in $a \to b$ in the orthogonal case and anti-symmetric in the symplectic case. We have also $\varepsilon_{ab}\varepsilon^{bc} = \delta_a^c$, $\varepsilon^{ab} = \varepsilon_{-a}\delta^{a,-b}$.

Let us define the L matrix as

$$L_{ab} = u\delta_{ab} - G_{ab} \tag{24}$$

where $G_{ab}, a, b = -n, -n+1, ..., -1, (0), 1, ..., n$ obey the Lie algebra relations of $so(2n(+1)), sp(n)$, respectively.

We consider the RLL relation in the form

$$R_{b_1 b_2}^{a_1 a_2}(u) L_{c_1}^{b_1}(u+v) L_{c_2}^{b_2}(v) = L_{b_2}^{a_2}(v) L_{b_1}^{a_1}(u+v) R_{c_1 c_2}^{b_1 b_2}(u), \tag{25}$$

and look for the conditions under which this relation is obeyed.

We obtain that the additional condition on the generators for allowing a linear L matrix reads in the uniform notation

$$\varepsilon_{b_1,b_2}[G^{a_1 b_1}, G^{a_2,b_2}]_+ = A\varepsilon^{a_1,a_2}. \tag{26}$$

If we choose the generators G^{ab} composed of the Clifford generators γ^a (13, 16) as

$$G^{ab} = \frac{1}{4}\varepsilon_{-a}[\gamma^a, \gamma^{-b}]_\mp \tag{27}$$

the additional condition (26) is fulfilled.

The Clifford algebra relation and its symplectic modification can be written uniformly as

$$[\gamma^a, \gamma^b]_\pm = 2\varepsilon^{ba}, \tag{28}$$

where we can choose the Clifford generators as the combinations of the basic Heisenberg generators, $\gamma^a|_{so} = \partial_{-a}^\theta + \theta_{\bar{a}}, \quad \gamma^a|_{sp} = \partial_a - \varepsilon_a x_{-a}$.

5. The spinorial Yang-Baxter operators

In general, for $L_i(u) = uI - G_i$ with $G^{ab} = \frac{1}{2}\gamma^{[ab]}$ (27), the RLL relation

$$R_{12}(u-v)L_1(u)L_2(v) = L_1(v)L_2(u)R_{12}(u-v) \tag{29}$$

can be considered as the defining relation for $R_{12}(u)$ which acts in the tensor product of two spinor representations. This relation results by separation of the dependence on $u+v$ from $u-v$ in the symmetry condition

$$[G_1 + G_2, R_{12}(u)] = 0,$$

and in the additional defining condition

$$u(R_{12}(u)G_2 - G_1 R_{12}(u)) - (R_{12}(u)G_1 G_2 - G_1 G_2 R_{12}(u)) = 0. \tag{30}$$

The symmetry condition implies that the spinorial R operator decomposes into the invariants composed from the Clifford generators in both tensor factors γ_1^a and γ_2^a,

$$\varepsilon_{a_1,a_1'}...\varepsilon_{a_k,a_k'}\mathcal{S}\{\gamma_1^{a_1}...\gamma_1^{a_k}\}\ \mathcal{S}\{\gamma_2^{a_1'}...\gamma_2^{a_k'}\} = \varepsilon_{A_k,A_k'}\gamma_1^{A_k}\ \gamma_2^{A_k'}. \tag{31}$$

The last form defines abbreviations in terms of which we write the ansatz

$$R(u) = \sum_k \varepsilon_{A_k,A_{k'}}\gamma_1^{A_k}\gamma_2^{A_k'}\frac{r_k(u)}{k!}. \tag{32}$$

The product of Clifford generators $\mathcal{S}\{\gamma_1^{a_1}...\gamma_1^{a_k}\}$ is anti-symmetrised in the orthogonal case and symmetrised in the symplectic case. Recall that the generators G_i^{ab} are composed of those γ_i^a in the anti-symmetrised or symmetrised way too (27).

In the defining condition (30) we encounter the multiplication of (anti-)symmetrised products. The transformation of such products into (anti-)symmetrised terms is conveniently done by using generating functions. We use auxiliary variables κ^a and their scalar product with the Clifford generators $(\kappa\gamma) = \varepsilon_{ab}\kappa^a\gamma^b$. They are anti-commuting (Grassmann) ones in the orthogonal case and commuting in the symplectic case.

$$\mathcal{S}\{\gamma^{a_1}...\gamma^{a_k}\} = \partial^{a_k}...\partial^{a_1}\ e^{(\kappa\gamma)}|_{\kappa=0} = \partial^{A_k}\ e^{(\kappa\gamma)}|_{\kappa=0},$$

$$\partial_a\kappa^b = \delta_a^b, \partial^a = \varepsilon^{ab}\partial_b, \partial^a\kappa^b = \varepsilon^{ab}, \varepsilon_{ab}\varepsilon^{bc} = \delta_a^c.$$

We illustrate the computations encountered here in the following example.

$$\gamma^{A_k}\ \gamma^{[ab]} = \partial_1^{A_k}\ e^{(\kappa_1\gamma)}\ \partial_2^b\partial_2^a e^{(\kappa_2\gamma)}|_{\kappa_1=\kappa_2=0} =$$

$$\partial_1^{A_k}\partial_2^b\partial_2^a\ e^{((\kappa_1+\kappa_2)\gamma)}\varepsilon^{(\kappa_1\kappa_2)}|_{\kappa_1=\kappa_2=0}.$$

Here we have used the Baker-Hausdorff formula and the generalised Clifford relation (28),

$$[(\kappa_1\gamma),(\kappa_2\gamma)] = \varepsilon_{ab}\varepsilon_{cd}\kappa_1^a\kappa_2^c[\gamma^b,\gamma^d]_\pm = 2\varepsilon^{db}\varepsilon_{ab}\varepsilon_{cd}\kappa_1^a\kappa_2^c =$$

$$2\varepsilon_{ac}\kappa_1^a\kappa_2^c = 2(\kappa_1\kappa_2).$$

Now we transform κ_1, κ_2 to $\kappa = \kappa_1 + \kappa_2, \kappa_2$. The derivatives with respect to the auxiliary κ variables then transform as $\partial_1 \to \partial, \partial_2 \to \partial + \partial_2$. This results in

$$\gamma^{A_k}\ \gamma^{[ab]} = \partial^{A_k}(\partial^b + \partial_2^b)(\partial^a + \partial_2^a)\ e^{(\kappa\gamma)}e^{(\kappa\kappa_2)}|_{\kappa=\kappa_2=0} =$$

$$\partial^{A_k}\{(\partial^b - \kappa^b)(\partial^a - \kappa^a) + \varepsilon^{ba}\}\ e^{(\kappa\gamma)}|_{\kappa=0}.$$

In this way we obtain that the additional defining condition (30) results in the relation for the coefficients of the expansion (32)

$$r_{k+2}(u) = \frac{u+k}{u-k\pm 2\beta_\pm}r_k(u). \tag{33}$$

The iteration goes in steps of 2, thus the spinorial R operator decomposes into the even and the odd parts, both obeying the defining relation independently, $R(u) = R^+(u) + R^-(u)$.

6. Discussion

We have considered the special representations (generators G_{ab}) of the orthogonal and symplectic algebras resulting in L operators of the form $L = uI - G$ obeying the YB relation with the fundamental R matrix. We have discussed the orthogonal and symplectic cases in analogy starting from the JS type representations of the general linear algebra.

In the Jordan-Schwinger case, the reduction of the algebra of $g\ell(2n(+1))$ to $so(2n(+1))$ or $sp(n)$ can be related to the elementary linear canonical transformation. This is the same transformation which connects the two fundamental representations of $g\ell(2n(+1))$ dual to each other.

In the resulting JS formulation of the orthogonal or symplectic generators a linear transformation of the underlying Heisenberg algebra generators leads to a separation in two terms which both obey the orthogonal or symplectic algebra and mutually commute. Thus in this form the algebra decomposes into two subalgebras. The generators of both subalgebras are found to be composed bilinearly of linear combinations of Heisenberg generators obeying the (quasi-) Clifford algebra relation.

These linear combinations of the underlying Heisenberg generators coincide with those obeying the condition of symmetry under the elementary canonical transformations.

The Clifford generators can be regarded equivalently as n Heisenberg pairs (creation and annihilation operators) fermionic in the orthogonal case and bosonic in the symplectic case. In the fermionic case we have the known spinor representation and in the symplectic case we find that the representation generated by n bosonic Heisenberg pairs is the appropriate counterpart of the spinorial one.

There are linear in the spectral parameter L operators in both cases, $L = Iu - G$, where $G^{a,b}$ are the generators of the orthogonal or symplectic algebra and obey additionally the condition (26). The generators bilinear in the Clifford generators obey this additional condition.

The Yang-Baxter R operator intertwining two spinorial representaions can be obtained in both orthogonal and symplectic cases in analogous way.

It can be checked that the fundamental R matrix (quadratic in u) can be reproduced by fusion including projection from the product of the spinorial L with its conjugate. More details about this and further results can be found in [7].

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