Topological Constants of Motion in Barotropic Fluid dynamics and Magnetohydrodynamics

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The formulation of a physical theory in a variational form allows one to use the continuous symmetries of the action in order to generate constants of motion through the Noether theorem. In particular topological constants of motion can be generated. The symmetry group related to the global helicity in fluid dynamics and cross helicity in magnetohydrodynamics [1,2] will be described. Yahalom & Lynden–Bell [1] introduced an Eulerian variational principle for magnetohydrodynamics. The variational principles were given in terms of six independent functions for non-stationary flows and three independent functions for stationary flows. This is less than the seven variables which appear in the standard equations of magnetohydrodynamics which are the magnetic field B the velocity field V and the density. Later [3] the number of needed functions was further reduced and it was shown that magnetohydrodynamics is mathematically equivalent to a four function field theory defined a by a Lagrangian. The four functions include two surfaces whose intersections consist the magnetic field lines, the part of the velocity field not defined by the commoving magnetic field and the density. The variational variables allow the derivation of local topological constants of motion [4,5] in terms of helicities per unit flux. The Lagrangians admit a group of relabeling [5] in the fluid dynamical case and diffeomorphism symmetry [6] in the case of magnetohydrodynamics. We will attempt to describe the significance of the constants of motion related to those symmetries.

References

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