On completeness of the Bethe ansatz for the XXX model

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I will consider the spin-1/2 XXX Heisenberg model on the finite chain with the periodic boundary condition. The completeness of the Bethe ansatz for this model claims the one-to-one correspondence between the SL(2)–invariant eigenspaces of the commuting transfer-matrices and solutions of a certain system of algebraic equations, and gives a formula for the eigenvectors. However, the claim is false in its naive form. I will describe the precise form of this correspondence and the refinement of the formula for the eigenvectors. Namely, the the SL(2)–invariant eigenspaces of the commuting transfer-matrices are in bijection with two-dimensional spaces of polynomials with a basis $p(u)$, $q(u)$, $\deg p < \deg q$, such that

$$p(u+i/2)q(u-\mathrm{i}/2) - p(u-\mathrm{i}/2)q(u+\mathrm{i}/2) = u^n\,, \tag{1}$$

where n is the length of the chain. The polynomial $p(u)$ of the smaller degree uniquely determines the two-dimensional space of polynomials, and the roots of $p(u)$ satisfy the standard Bethe ansatz equations. To obtain eigenvectors in some cases, one has to consider an inhomogeneous XXX model, replacing u^n in the right-hand side of equation (1) by an arbitrary polynomial of degree n, and then consider the limit to the homogeneous model.