Group theoretical approach to the quantum theory of an interacting particle

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The basic problems in establishing the Quantum Theory of a specific physical system are:

(1) to single out, modulo unitary isomorphism, the specific Hilbert space $\mathcal{H}$ for the specific theory,

(2) to concretely identify, among the self-adjoint operators of $\mathcal{H}$, which specific operator is the representative of each relevant observable;

(3) to determine the specific form of the hamiltonian operator H ruling over the dynamics.

Group theoretical methods, developed in particular by E.P. Wigner and G.W. Mackey, provided these problems with very satisfactory answers. If the physical system is a free particle, then Galilei's group G is a group of symmetry transformations for the system. By making use of Wigner's theorem on the representation of symmetries and of Mackey's imprimitivity theorem all items (1), (2) and (3) for this specific quantum theory are deductively fulfilled. Unfortunately, if the system is not isolated, generally Galilei's transformations are not symmetries, so that neither Wigner's theorem nor Mackey's theorem apply. The approaches to the Quantum Theory of an interacting particle present in the literature are not formulated with the epistemological soundness of the group theoretical approach so successful for the free particle specific case.

In this work we prove that a group theoretical approach can be pursued also for an interacting particle, with results which share the logical, formal and conceptual correctness of the free particle case [1]. In particular, we prove that the electromagnetic interaction is completely characterized in terms of covariance properties of the interaction with respect to Galilei's group.

References

[1] Giuseppe Nistico A group theoretical approach to quantum theory of the interaction for a non-relativistic spin-0 particle arXiv:1411.4127