Higher Sugawara operators and Gaudin Hamiltonians

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We will discuss recent constructions of higher Sugawara operators for the affine Kac-Moody algebras. For each simple Lie algebra $\mathfrak{g}$ consider the corresponding vacuum module $V_{\mathrm{cri}}(\mathfrak{g})$ over $\widehat{\mathfrak{g}}$ at the critical level. The $\mathfrak{g}[t]$-invariants of the vacuum module form a commutative associative algebra whose structure was described by a theorem of B. Feigin and E. Frenkel (1992). We give explicit formulas for generators of the algebra of invariants associated with the Lie algebras $\mathfrak{g}$ of classical types. The construction relies on the Schur–Weyl duality involving the Brauer algebra, and the generators are expressed as weighted traces over tensor spaces. This leads to explicit constructions of commutative subalgebras of the universal enveloping algebras $\mathrm{U}(\mathfrak{g})$, and to higher order Hamiltonians in the Gaudin model associated with each Lie algebra $\mathfrak{g}$. Moreover, we use the work of Feigin, Frenkel and Reshetikhin (1994) to calculate the eigenvalues of these Gaudin Hamiltonians on the Bethe vectors in an explicit form.