Classification of evolution equations with trivial and nontrivial $\rho^{(3)}$: An Application of "Level Grading"

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In this study we used the "formal symmetry" method of Mikhailov–Shabat–Sokolov as an integrability test which is based on the remark that if the evolution equation admits a recursion operator, than its expansion as a formal series satisfies an operator equation that has to be solved in the class of local functions. This requirement gives a sequence of conserved density conditions, called the "canonical conserved densities" $\rho^{(i)}$. We define a new grading, that we call the "level grading", on the algebra of polynomials generated by the derivatives u_{k+i} over the ring $\mathcal{K}^{(k)}$ of $\mathcal{C}^\infty$ functions of $x, t, u, u_1, \ldots, u_k$, where $u_j = \frac{\partial^j u}{\partial x^j}$. This grading has the property that the total derivative and the integration by parts with respect to x are filtered algebra maps. This crucial property allows to perform conserved density computations for each level separately, starting from the higher levels that give simpler equations. In addition, if u satisfies the evolution equation $u_t = F[u]$ where F is a polynomial of order $m = k + p$ and of level p, then the total derivative with respect to t, D_t, is also a filtered algebra map. Furthermore, if the separant $\frac{\partial F}{\partial u_m}$ belongs to $\mathcal{K}^{(k)}$, then the canonical densities $\rho^{(i)}$ are polynomials of level $2i+1$ and $D_t\rho^{(i)}$ is of level $2i+1+m$. We use the properties of level grading to obtain a preliminary classification of scalar evolution equations of orders $m = 7, 9, 11, 13, 15, 17$ up to their dependence on x, t, u, u_1 and u_2. We also study those evolution equations for which the canonical density $\rho^{(3)}$ is trivial.