Family of N-dimensional superintegrable systems and quadratic algebra structures

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Classical and quantum superintegrable systems have a long story and they possess more integrals of motion than degrees of freedom. They have many attractive properties, wide applications in modern physics and connected to many domains in pure and applied mathematics. We introduce new families of superintegrable Kepler–Coulomb systems with non-central terms and superintegrable Hamiltonians with double singular oscillators of type $(n, N-n)$ in N-dimensional Euclidean space. We construct quadratic algebra structures $Q(3) \oplus \mathrm{so}(N-1)$, $Q(3) \oplus \mathrm{so}(n) \oplus \mathrm{so}(N-n)$ and the cubic Casimir operators of these models. We show these algebras can be realized in terms of deformed oscillator algebras and obtain the finite dimensional unitary representations. We present algebraic derivations of the degenerate energy spectra of these systems and relate with the physical spectra obtained from the separation of variables. We will also discuss further generalizations to monopole interactions, more complicated quadratic algebra structures and use of recurrence approached to the superintegrable systems.