

Functions of Positive Type and Open Quantum Systems

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Functions of positive type on locally compact abelian groups, defined as positive functionals on group algebras, play a remarkable role in probability theory and in classical statistical mechanics since, according to Bochner's theorem, they are characterized as being (up to normalization) Fourier transforms of probability measures. A similar result holds in the quantum setting as well, where a generalized kind of functions of positive type on phase space – the so-called (continuous) functions of quantum positive type – can be obtained, via the Fourier-Plancherel transform, from the Wigner quasi-probability distributions. We will argue that, as in the classical case, the notion of function of quantum positive type is group-theoretical. Next, exploiting the properties of functions of positive type on phase space, both classical and quantum, we will introduce a class of semi-groups of operators that describe the evolution of certain open quantum systems which are of interest in quantum information science. Finally, a suitable extension of this framework to generalized phase spaces that are relevant for current applications will be outlined.