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Note

Corrigendum: "On Brlek–Reutenauer conjecture"

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ABSTRACT

Bašić (2012) in [1] pointed to a gap in the proof of Corollary 5.10 in Balková et al. (2011) [2] related to the Brlek–Reutenauer conjecture. In this corrigendum, we correct the proof and show that the corollary remains valid.

1. Corrigendum

Brlek and Reutenauer conjectured that any infinite word $\mathbf{u}$ with language closed under reversal satisfies the equality $2D(\mathbf{u}) = \sum_{n=0}^{+\infty} T_{\mathbf{u}}(n)$ in which $D(\mathbf{u})$ denotes the defect of $\mathbf{u}$ and $T_{\mathbf{u}}(n)$ denotes $\mathcal{C}_{\mathbf{u}}(n+1) - \mathcal{C}_{\mathbf{u}}(n) + 2 - \mathcal{P}_{\mathbf{u}}(n+1) - \mathcal{P}_{\mathbf{u}}(n)$, where $\mathcal{C}_{\mathbf{u}}$ and $\mathcal{P}_{\mathbf{u}}$ are the factor and palindromic complexity of $\mathbf{u}$, respectively. In [2], we proved their conjecture for uniformly recurrent words. In the same paper in the section Open problems, we discussed various statements related to the Brlek–Reutenauer conjecture that can be stated for infinite words which are not uniformly recurrent. One of the statements was the following implication.

Corollary 1 (*Corollary 5.10 in [2]*). *Let $\mathbf{u}$ be an infinite word with the language closed under reversal. Then we have*

$$D(\mathbf{u}) < +\infty \;\Rightarrow\; \sum_{n=0}^{+\infty} T_n(\mathbf{u}) < +\infty.$$

For the proof of Corollary 5.10 we used Theorem 5.7.

Theorem 2 (*Theorem 5.7 in [2]*). *Let $\mathbf{u}$ be an infinite word with language closed under reversal and containing infinitely many palindromes. The following statements are equivalent.*

1. *The defect of $\mathbf{u}$ is finite.*
2. *$\mathbf{u}$ has only finitely many oddities.*
3. *There exists an integer H such that the longest palindromic suffix of any factor w of length $|w| \geq H$ occurs in w exactly once.*

In [1], the author found a counterexample to the implication 1. $\Rightarrow$ 3.
Theorem 2 stays valid if we replace 3. with the following statement:

3'. There exists an integer H such that the longest palindromic suffix of any prefix w of length $|w| \geq H$ occurs in w exactly once.

Proof. The equivalence 1. $\Leftrightarrow$ 3.' follows from the fact that $D(\mathbf{u})$ is equal to the number of prefixes of $\mathbf{u}$ whose longest palindromic suffix is not unioccurrent. □

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Even this weaker theorem enables us to prove Corollary 1 as follows.

Lemma 3. *Let $\mathbf{u}$ be an infinite word. If there exists an integer H such that any prefix p of $\mathbf{u}$ of length $|p| > H$ has a unioccurrent longest palindromic suffix, then for any factor w such that $|w| \geq H$, any factor of $\mathbf{u}$ longer than $|w|$ beginning in w or $\overline{w}$ and ending in w or $\overline{w}$, with no other occurrences of w or $\overline{w}$, is a palindrome.*

Proof. Let us show the statement by contradiction. Suppose there exists a factor $w \in \mathcal{L}(\mathbf{u})$ such that $|w| \geq H$ and there exists a non-palindromic factor of $\mathbf{u}$ beginning in w or $\overline{w}$ and ending in w or $\overline{w}$, with no other occurrences of w or $\overline{w}$. Let us find the first non-palindromic factor of this form in $\mathbf{u}$ and let us denote it r. Let p be the prefix of $\mathbf{u}$ ending in the first occurrence of r in $\mathbf{u}$, i.e., $p = tr$ for some word t and r is unioccurrent in p. Denote by s the longest palindromic suffix of p. By the assumption, s is unioccurrent in p. No matter how long the suffix s is, we will obtain a contradiction.

(1) If $|s| \leq |w|$, then we have a contradiction to the unioccurrence of s.
(2) If $|r| > |s| > |w|$, then we can find at least 3 occurrences of w or $\overline{w}$ in r which is a contradiction to the form of r.
(3) The equality $|r| = |s|$ contradicts the fact that we supposed r to be non-palindromic.
(4) Finally, if $|r| < |s|$, then there is an occurrence of the mirror image of r which is a non-palindromic factor having the same properties as r which occurs before r and contradicts the choice of p. □

Proof of Corollary 1. We will make use of Lemma 2.8 in [2]. Assume $D(\mathbf{u}) < +\infty$. By the implication 1. ⇒ 3'. in the corrected Theorem 2, there exists an integer H such that any prefix p of $\mathbf{u}$ of length $|p| > H$ has a unioccurrent longest palindromic suffix. Let us show for any $n > H$ that the assumptions of Lemma 2.8 are satisfied.

We have to show two properties of $G_n(\mathbf{u})$ for any $n > H$.

(1) Any loop in $G_n(\mathbf{u})$ is a palindrome.

Since any loop e in $G_n(\mathbf{u})$ at a vertex $(w, \overline{w})$ is a word beginning in w or $\overline{w}$ and ending in w or $\overline{w}$, with no other occurrences of w or $\overline{w}$, the loop e is a palindrome by Lemma 3.

(2) The graph obtained from $G_n(\mathbf{u})$ by removing loops is a tree.

Or equivalently, we show that in $G_n(\mathbf{u})$ there exists a unique path between any two different vertices $(w', \overline{w'})$ and $(w'', \overline{w''})$. Let p be a factor of $\mathbf{u}$ such that w' or $\overline{w'}$ is its prefix, w'' or $\overline{w''}$ is its suffix and p has no other occurrences of w', $\overline{w'}$, w'', $\overline{w''}$. Let v be a factor starting in p, ending in w' or $\overline{w'}$ and containing no other occurrences of w' or $\overline{w'}$. By Lemma 3 the factor v is a palindrome, thus $\overline{p}$ is a suffix of v. It is then a direct consequence of the construction of v that the next factor with the same properties as p, i.e., representing a path in the undirected graph $G_n(\mathbf{u})$ between w' and w'', which occurs in $\mathbf{u}$ after p, is $\overline{p}$. This shows that there is only one such path.

Consequently, Lemma 2.8 implies that $T_n(\mathbf{u}) = 0$ for any $n > H$. □

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References

[1] B. Bašić, A note on the paper "On Brlek–Reutenauer conjecture", Theoret. Comput. Sci. 448 (2012) 94–96.
[2] L. Balková, E. Pelantová, Š Starosta, On Brlek–Reutenauer conjecture, Theoret. Comput. Sci. 412 (2011) 5649–5655.