BÍLÁ **Vzorové řešení – varianta A** 17. 12. 2019

(1) • $A \in \mathcal{L}(\mathbb{R}^3, \mathbb{R}^2)$

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$\forall \alpha \in \mathbb{R}\ \forall \vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3\ \forall \vec{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \in \mathbb{R}^3 \quad A(\alpha\vec{x}+\vec{y}) = \alpha A\vec{x} + A\vec{y}$,

protože $\alpha\vec{x}+\vec{y} = \begin{pmatrix} \alpha x_1 + y_1 \\ \alpha x_2 + y_2 \\ \alpha x_3 + y_3 \end{pmatrix} \quad A(\alpha\vec{x}+\vec{y}) = \begin{pmatrix} -3(\alpha x_1 + y_1) \\ \alpha x_2 + y_2 \end{pmatrix}$

$$\alpha A\vec{x} + A\vec{y} = \alpha\begin{pmatrix} -3x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} -3y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \alpha(-3x_1) + (-3y_1) \\ \alpha x_2 + y_2 \end{pmatrix}$$

• A není prosté, $A\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = A\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow$ A není prosté

30% tudíž ani monomorfní

(2) ${}^{\mathcal{E}_2}A^{\mathcal{Y}} = {}^{\mathcal{X}}A^{\mathcal{Y}}\,{}^{\mathcal{E}_2}I^{\mathcal{X}} = ({}^{\mathcal{E}_3}I^{\mathcal{Y}}\,{}^{\mathcal{X}}A^{\mathcal{E}_3})\,{}^{\mathcal{E}_2}I^{\mathcal{X}} = \begin{pmatrix} 2 & 1 & -2 \\ -1 & -1 & 2 \\ 0 & 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 2 & 2 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 1 & -2 \\ -1 & -1 & 2 \\ 0 & 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ -3 & -2 \\ 2 & 1 \end{pmatrix}$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & +1 & 0 & +1 & -1 \end{array}\right)$$

${}^{\mathcal{E}_3}I^{\mathcal{Y}} = ((\vec{e}_1)_{\mathcal{Y}}\,(\vec{e}_2)_{\mathcal{Y}}\,(\vec{e}_3)_{\mathcal{Y}}) = \begin{pmatrix} 2 & 1 & -2 \\ -1 & -1 & 2 \\ 0 & 1 & -1 \end{pmatrix}$

${}^{\mathcal{E}_2}I^{\mathcal{X}} = ((\vec{e}_1)_{\mathcal{X}}\,(\vec{e}_2)_{\mathcal{X}}) = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$

správný postup 50%
výsledek 50%

(3) $AB \in \mathcal{L}(\mathbb{R}^2)$

(a) ${}^{\mathcal{E}_2}(AB)^{\mathcal{E}_2} = {}^{\mathcal{E}_3}A^{\mathcal{E}_2}\,{}^{\mathcal{E}_2}B^{\mathcal{E}_3} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

50%

(b) $h(AB) = h({}^{\mathcal{E}}(AB)) = 2$

50% $d(AB) = \dim\mathbb{R}^2 - h(AB) = 0 \Rightarrow \ker(AB) = \left\{\begin{pmatrix} 0 \\ 0 \end{pmatrix}\right\}$

(4) $\overset{-1}{B}(\vec{b}) = \emptyset$, nebo $\overset{-1}{B}(\vec{b}) = \vec{a} + \ker B$

$$\left(\begin{array}{ccc|c} 6 & 0 & -3 & 3 \\ 4 & 0 & -2 & 2 \\ 2 & 0 & -1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

(a) partikulár. řeš.: $B\vec{a} = \vec{b} \Leftrightarrow {}^{\mathcal{X}}B^{\mathcal{X}}(\vec{a})_{\mathcal{X}} = (\vec{b})_{\mathcal{X}}$

60% $(\vec{a})_{\mathcal{X}} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \Rightarrow \vec{a} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

ker B: $B\vec{x} = \vec{o} \Leftrightarrow {}^{\mathcal{X}}B^{\mathcal{X}}(\vec{x})_{\mathcal{X}} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$(\vec{x})_{\mathcal{X}} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, (\vec{y})_{\mathcal{X}} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \Rightarrow \vec{x} = \begin{pmatrix} -1 \\ 0 \\ -2 \end{pmatrix}, \vec{y} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + 2\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$

$h(B) = h({}^{\mathcal{X}}B^{\mathcal{X}}) = 1 \Rightarrow d(B) = 3 - 1 = 2$

$\ker B = \left[\begin{pmatrix} +1 \\ 0 \\ +2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}\right]_\lambda$

(b) partikulár. řeš.: potřebujeme $(\vec{b})_{\mathcal{X}}$

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$\overset{-1}{B}(\vec{b}) = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + \ker B$

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & 3 \\ -1 & 0 & 1 & 2 \\ 0 & -2 & -1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 0 & 3 \\ 0 & -1 & 1 & 5 \\ 0 & 0 & -3 & -9 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 0 & 3 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 1 & 3 \end{array}\right) \qquad (\vec{b})_* = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$$

$$B\vec{a} = \vec{b} \Leftrightarrow {}^*B^*(\vec{a})_* = (\vec{b})_*$$

$$\left(\begin{array}{ccc|c} 6 & 0 & -3 & 1 \\ 4 & 0 & -2 & -2 \\ 2 & 0 & -1 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

nemá řešení,

tj. $\overset{-1}{B}(\vec{b}) = \emptyset$

ŽLUTÁ *Vzorové řešení – varianta B* 17. 12. 2019

① (a) $AB \in \mathcal{L}(\mathbb{R}^2)$

50% $\quad {}^{\varepsilon_2}(AB) = {}^{\varepsilon_3}A^{\varepsilon_2}\,{}^{\varepsilon_2}B^{\varepsilon_3} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$

(b) $h(AB) = h\left({}^{\varepsilon_2}(AB)^{\varepsilon_2}\right) = 1$

50% $\quad d(AB) = \dim \mathbb{R}^2 - h(AB) = 1$

$(AB)\vec{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \iff {}^{\varepsilon_2}(AB)(\vec{x})_{\varepsilon_2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \vec{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \ker(AB) = \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right]_\lambda$

② $\overset{-1}{B}(\vec{b}) = \emptyset$, nebo $\overset{-1}{B}(\vec{b}) = \vec{a} + \ker B$

(a) partikulár. řeš.: $B\vec{a} = \vec{b} \iff {}^{y}B^{y}(\vec{a})_y = (\vec{b})_y$

40% $\quad \left(\begin{array}{ccc|c} 0 & 2 & -1 & -6 \\ 1 & -1 & 2 & -4 \\ -1 & 1 & -3 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 2 & -4 \\ 0 & 2 & -1 & -6 \\ 0 & 0 & +1 & +6 \end{array}\right) \quad (\vec{b})_y = \begin{pmatrix} -16 \\ 0 \\ 6 \end{pmatrix}$

$\left(\begin{array}{ccc|c} 6 & 0 & -3 & -16 \\ 4 & 0 & -2 & 0 \\ 2 & 0 & -1 & 6 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 0 & -1 & 6 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$

nemá řeš.

$\overset{-1}{B}(\vec{b}) = \emptyset$

(b) partikulár. řeš.: $B\vec{a} = \vec{b} \iff {}^{y}B^{y}(\vec{a})_y = (\vec{b})_y$

60% $\quad \left(\begin{array}{ccc|c} 6 & 0 & -3 & -6 \\ 4 & 0 & -2 & -4 \\ 2 & 0 & -1 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 0 & -1 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad (\vec{a})_y = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \vec{a} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

$\ker B$: $B\vec{x} = \vec{o} \iff {}^{y}B^{y}(\vec{x})_y = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad (\vec{x})_y = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, (\vec{y})_y = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \Rightarrow$

$h(B) = h({}^{y}B^{y}) = 1 \Rightarrow d(B) = 3 - 1 = 2 \qquad \Rightarrow \vec{x} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \vec{y} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + 2\begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \\ -7 \end{pmatrix}$

$\overset{-1}{B}(\vec{b}) = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + \left[\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -5 \\ 7 \end{pmatrix}\right]_\lambda \qquad \ker B = \left[\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 5 \\ -7 \end{pmatrix}\right]_\lambda$

③ • $A \notin \mathcal{L}(\mathbb{R}^3, \mathbb{R}^2)$ $\quad \underbrace{A\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}}_{=\begin{pmatrix} 1 \\ 1 \end{pmatrix}} \neq \underbrace{(-1)A\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}_{=(-1)\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}}$, což je spor s homogenitou

70%

• A není monomorfní – není ani lineární, ani prosté

30% (např. $A\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = A\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$), a už při absenci jedné z těchto vlastností není monomorfní

④ $${}^{\mathcal{E}_2}A^{\mathcal{Y}} = {}^{\mathcal{X}}A^{\mathcal{Y}}\;{}^{\mathcal{E}_2}I^{\mathcal{X}} = {}^{\mathcal{E}_3}I^{\mathcal{Y}}\;{}^{\mathcal{X}}A^{\mathcal{E}_3}\;{}^{\mathcal{E}_2}I^{\mathcal{X}} = \begin{pmatrix}2&1&-2\\-1&-1&2\\0&1&-1\end{pmatrix}\begin{pmatrix}1&1\\2&0\\-1&0\end{pmatrix}\begin{pmatrix}1&1\\1&2\end{pmatrix} = \begin{pmatrix}6&2\\-5&-1\\3&0\end{pmatrix}\begin{pmatrix}1&1\\1&2\end{pmatrix} = \begin{pmatrix}8&10\\-6&-7\\3&3\end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc}1&1&0&1&0&0\\1&2&2&0&1&0\\1&2&1&0&0&1\end{array}\right) \sim \left(\begin{array}{ccc|ccc}1&1&0&1&0&0\\0&1&2&-1&1&0\\0&0&+1&0&+1&-1\end{array}\right) \sim \left(\begin{array}{ccc|ccc}1&0&0&2&1&-2\\0&1&0&-1&-1&2\\0&0&1&0&1&-1\end{array}\right)$$

$${}^{\mathcal{E}_3}I^{\mathcal{Y}} = \begin{pmatrix}2&1&-2\\-1&-1&2\\0&1&-1\end{pmatrix} = \left((\vec{e_1})_{\mathcal{Y}}\,(\vec{e_2})_{\mathcal{Y}}\,(\vec{e_3})_{\mathcal{Y}}\right)$$

$$\left(\begin{array}{cc|cc}2&-1&1&0\\-1&1&0&1\end{array}\right) \sim \left(\begin{array}{cc|cc}-1&1&0&1\\0&1&1&2\end{array}\right)$$

$${}^{\mathcal{E}_2}I^{\mathcal{X}} = \left((\vec{e_1})_{\mathcal{X}}\,(\vec{e_2})_{\mathcal{X}}\right) = \begin{pmatrix}1&1\\1&2\end{pmatrix}$$

správný postup 50%

výsledek 50%

MODRÁ   Vzorové řešení – varianta C   17. 12. 2019

1.

$${}^{x}A^{\varepsilon_3} = {}^{y}I^{\varepsilon_3}\,{}^{x}A^{y} = {}^{y}I^{\varepsilon_3}\,{}^{\varepsilon_2}A^{y}\,{}^{x}I^{\varepsilon_2} = \begin{pmatrix}1&1&0\\1&2&2\\1&2&1\end{pmatrix}\begin{pmatrix}1&1\\2&2\\-1&0\end{pmatrix}\begin{pmatrix}2&-1\\3&1\end{pmatrix} = \begin{pmatrix}3&3\\3&5\\4&5\end{pmatrix}\begin{pmatrix}2&-1\\3&1\end{pmatrix} = \begin{pmatrix}15&0\\21&2\\23&1\end{pmatrix}$$

$${}^{y}I^{\varepsilon_3} = \left((\vec{y_1})_{\varepsilon_3}\ (\vec{y_2})_{\varepsilon_3}\ (\vec{y_3})_{\varepsilon_3}\right) = \begin{pmatrix}1&1&0\\1&2&2\\1&2&1\end{pmatrix}$$

$${}^{x}I^{\varepsilon_2} = \left((\vec{x_1})_{\varepsilon_2}\ (\vec{x_2})_{\varepsilon_2}\right) = \begin{pmatrix}2&-1\\3&1\end{pmatrix}$$

správný postup 50%

výsledek 50%

2. (a) $BA \in \mathcal{L}(\mathbb{R}^3)$

50% $${}^{\varepsilon_3}(BA) = {}^{\varepsilon_2}B^{\varepsilon_3}\,{}^{\varepsilon_3}A^{\varepsilon_2} = \begin{pmatrix}-1&1\\1&1\\1&-1\end{pmatrix}\begin{pmatrix}1&1&0\\1&0&1\end{pmatrix} = \begin{pmatrix}0&-1&1\\2&1&1\\0&1&-1\end{pmatrix}$$

(b) $h(BA) = h\left({}^{\varepsilon_3}(BA)\right) = 2$ $\begin{pmatrix}0&-1&1\\2&1&1\\0&1&-1\end{pmatrix} \sim \begin{pmatrix}2&1&1\\0&1&-1\\0&0&0\end{pmatrix}$

50% $d(BA) = 3 - h(BA) = 1$

$(BA)\vec{x} = \begin{pmatrix}0\\0\\0\end{pmatrix} = {}^{\varepsilon_3}(BA)\cdot(\vec{x})_{\varepsilon_3}$ $\ker(BA) = \left[\begin{pmatrix}-1\\1\\1\end{pmatrix}\right]_\lambda$

3. • $A \notin \mathcal{L}(\mathbb{R}^3, \mathbb{R}^2)$ $A\begin{pmatrix}-1\\0\\0\end{pmatrix} \neq (-1)\,A\begin{pmatrix}1\\0\\0\end{pmatrix}$, tj. A není homogenní

70%

$$\begin{array}{ccc} \| & & \| \\ \begin{pmatrix}-3\\0\end{pmatrix} & & (-1)\begin{pmatrix}-3\\0\end{pmatrix} = \begin{pmatrix}3\\0\end{pmatrix}\end{array}$$

• A není epimorfní, – A není ani lineární, ani na $\mathbb{R}^2$

30% (např. $\forall \vec{x} \in \mathbb{R}^3\ A\vec{x} \neq \begin{pmatrix}1\\0\end{pmatrix}$, protože 1. složka $A\vec{x}$ je vždy ≤ 0),
a absence jedné z těchto vlastností stačí, aby A nebylo epimorfní

4. $\overset{-1}{B}(\vec{b}) = \emptyset$, nebo $\overset{-1}{B}(\vec{b}) = \vec{a} + \ker B$

(a) partikulární řešení: $B\vec{a} = \vec{b} \iff {}^{x}B^{y}(\vec{a})_x = (\vec{b})_y$

40% $$\left(\begin{array}{ccc|c}0&2&1&3\\1&-1&2&2\\-1&4&1&1\end{array}\right) \sim \left(\begin{array}{ccc|c}1&-1&2&2\\0&3&3&3\\0&2&1&3\end{array}\right) \sim \left(\begin{array}{ccc|c}1&-1&2&2\\0&1&1&1\\0&0&-1&1\end{array}\right)$$

$(\vec{b})_y = \begin{pmatrix}6\\2\\-1\end{pmatrix}$

$$\left(\begin{array}{ccc|c}0&6&-3&6\\0&4&-2&2\\0&2&-1&-1\end{array}\right) \sim \left(\begin{array}{ccc|c}0&2&-1&-1\\0&0&0&1\\0&0&0&0\end{array}\right)$$

nemá řeš.

$\overset{-1}{B}(\vec{b}) = \emptyset$

(b) partikulár. řeš.: $B\vec{a} = \vec{b} \Leftrightarrow {}^{x}B^{y} \cdot (\vec{a})_x = (\vec{b})_y$ $\left(\begin{array}{ccc|c} 0 & 6 & -3 & 3 \\ 0 & 4 & -2 & 2 \\ 0 & 2 & -1 & 1 \end{array}\right) \sim$

60 %

$(\vec{a})_x = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \Rightarrow \vec{a} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$ $\sim \left(\begin{array}{ccc|c} 0 & 2 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

ker B: $B\vec{x} = \vec{o} \Leftrightarrow {}^{x}B^{y} \cdot (\vec{x})_x = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ $(\vec{x})_x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, (\vec{y})_x = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$

$d(B) = -h(B) + 3 = -1 + 3 = 2$

$\overset{\|}{h({}^{x}B^{y})}$

$\Rightarrow \vec{x} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \vec{y} = \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} + 2\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \\ -4 \end{pmatrix}$

$\ker B = \left[\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 4 \\ -4 \end{pmatrix}\right]_\lambda$

$B^{-1}(\vec{b}) = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + \left[\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 4 \\ -4 \end{pmatrix}\right]_\lambda$

ZELENÁ   Vzorové řešení – varianta D   17. 12. 2019

1. $B^{-1}(\vec b) = \emptyset$, nebo $B^{-1}(\vec b) = \vec a + \ker B$

(a) partikulár. řeš.: $B\vec a = \vec b \iff {}^yB^x \cdot (\vec a)_y = (\vec b)_x$

60%

$$\left(\begin{array}{ccc|c} 0 & 6 & -3 & 3 \\ 0 & 4 & -2 & 2 \\ 0 & 2 & -1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 2 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$(\vec a)_y = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \Rightarrow \vec a = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$$

$\ker B$: $B\vec x = \vec o \iff {}^yB^x \cdot (\vec x)_y = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$(\vec x)_y = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix},\ (\vec y)_y = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \Rightarrow$$

$$d(B) = 3 - h(B) = 3 - h({}^yB^x) = 3 - 1 = 2$$

$$\Rightarrow \vec x = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix},\ \vec y = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} + 2\begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ -2 \end{pmatrix}$$

$$B^{-1}(\vec b) = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + \left[\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ -2 \end{pmatrix}\right]_\lambda \qquad \ker B = \left[\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ -2 \end{pmatrix}\right]_\lambda$$

(b) partikulár. řeš.: $B\vec a = \vec b \iff {}^yB^x \cdot (\vec a)_y = (\vec b)_x$

40%

$$\left(\begin{array}{ccc|c} 1 & -1 & 3 & 3 \\ -1 & 2 & 3 & 2 \\ 2 & -2 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 3 & 3 \\ 0 & 1 & 6 & 5 \\ 0 & 0 & -5 & -5 \end{array}\right) \qquad (\vec b)_x = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 0 & 6 & -3 & -1 \\ 0 & 4 & -2 & -1 \\ 0 & 2 & -1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 2 & -1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

nemá řeš., $B^{-1}(\vec b) = \emptyset$

2. ${}^xA^{\varepsilon_3} = {}^{\varepsilon_2}A^{\varepsilon_3}\,{}^xI^{\varepsilon_2} = {}^yI^{\varepsilon_3}\,{}^{\varepsilon_2}A^y\,{}^xI^{\varepsilon_2} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 1 & 2 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 2 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 0 & 5 \\ -1 & 5 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 11 & 2 \\ 15 & 5 \\ 13 & 6 \end{pmatrix}$

$${}^yI^{\varepsilon_3} = \left((\vec y_1)_{\varepsilon_3}\ (\vec y_2)_{\varepsilon_3}\ (\vec y_3)_{\varepsilon_3}\right) = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

$${}^xI^{\varepsilon_2} = \left((\vec x_1)_{\varepsilon_2}\ (\vec x_2)_{\varepsilon_2}\right) = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$$

Správný postup 50%
výsledek 50%

3. (a) $BA \in \mathcal{L}(\mathbb{R}^3)$

50% ${}^{\varepsilon_3}(BA) = {}^{\varepsilon_2}B^{\varepsilon_3} \cdot {}^{\varepsilon_3}A^{\varepsilon_2} = \begin{pmatrix} -1 & 1 \\ 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \end{pmatrix} = \begin{pmatrix} -1 & -2 & 2 \\ 1 & 0 & 2 \\ 1 & 2 & -2 \end{pmatrix}$

(b) 50% $h(BA) = h({}^{\varepsilon_3}(BA)) = 2$ $d(BA) = \dim\mathbb{R}^3 - h(BA) = 1$

$$\begin{pmatrix} -1 & -2 & 2 \\ 1 & 0 & 2 \\ 1 & 2 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & -2 \\ 0 & -2 & 4 \\ 0 & 0 & 0 \end{pmatrix}$$

$(BA)\vec x = \vec o \overset{\varepsilon_3}{=} (BA) \cdot (\vec x)_{\varepsilon_3}$ $\ker B = \left[\begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}\right]_\lambda$

(4.) • $A \in \mathcal{L}(\mathbb{R}^3, \mathbb{R}^2)$

70%

$\forall \alpha \in \mathbb{R} \quad \forall \vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 \quad \forall \vec{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \in \mathbb{R}^3 \quad A(\alpha\vec{x} + \vec{y}) = \alpha A\vec{x} + A\vec{y}$

$$\alpha\vec{x} + \vec{y} = \begin{pmatrix} \alpha x_1 + y_1 \\ \alpha x_2 + y_2 \\ \alpha x_3 + y_3 \end{pmatrix} \qquad A(\alpha\vec{x} + \vec{y}) = \begin{pmatrix} 0 \\ \alpha x_2 + y_2 + 2(\alpha x_3 + y_3) \end{pmatrix} \;\|$$

$$\alpha A\vec{x} + A\vec{y} = \alpha \begin{pmatrix} 0 \\ x_2 + 2x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ y_2 + 2y_3 \end{pmatrix} = \begin{pmatrix} 0 \\ \alpha(x_2 + 2x_3) + y_2 + 2y_3 \end{pmatrix}$$

• A není epimorfní, je sice lineární, ale není „na $\mathbb{R}^2$"

30%

(např. $\forall \vec{x} \in \mathbb{R}^3 \quad A\vec{x} \neq \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, protože 1. složka $A\vec{x}$ je nulová)