1. kontrolní test A

(1.) (a) $V \subset\subset \mathbb{C}^3$, tudíž V je vektorový prostor nad $\mathbb{C}$

70%

$V \neq \emptyset$ (např. $\begin{pmatrix}0\\0\\0\end{pmatrix} \in V$)

V je uzavřená na $\oplus$ a $\odot$:

$\forall \vec{x} = \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} \in V \quad \forall \vec{y} = \begin{pmatrix}y_1\\y_2\\y_3\end{pmatrix} \in V \quad \forall \alpha \in \mathbb{C}$

$\vec{x} \oplus \vec{y} = \begin{pmatrix}x_1+y_1\\x_2+y_2\\x_3+y_3\end{pmatrix} \in V$

platí totiž $x_1+x_2=0$, tudíž
$y_1+y_2=0$

$(x_1+y_1)+(x_2+y_2) = \underbrace{(x_1+x_2)}_{0} + \underbrace{(y_1+y_2)}_{0} = 0$

$\alpha \odot \vec{x} = \begin{pmatrix}\alpha_1 x_1\\ \alpha_2 x_2\\ \alpha_3 x_3\end{pmatrix} \in V$

platí totiž $x_1+x_2=0$, tudíž
$\alpha x_1 + \alpha x_2 = \alpha \underbrace{(x_1+x_2)}_{0} = 0$

30%

(b) $V \not\subset\subset \mathbb{C}^3$, tudíž V není vektor. prostor nad $\mathbb{C}$

možné argumenty, stačí jeden z nich

- V neobsahuje $\vec{0} = \begin{pmatrix}0\\0\\0\end{pmatrix}$
- V není uzavřená na $\oplus$, např. $\begin{pmatrix}1\\0\\0\end{pmatrix} \in V$, $\begin{pmatrix}0\\1\\0\end{pmatrix} \in V$, ale $\begin{pmatrix}1\\1\\0\end{pmatrix} \notin V$
- V není uzavřená na $\odot$, např. $\begin{pmatrix}1\\0\\0\end{pmatrix} \in V$, ale $2 \odot \begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}2\\0\\0\end{pmatrix} \notin V$

(2.) Pracujeme se souřadnicemi ve standardní bázi.

$$\begin{pmatrix}1&1&0&2&1\\2&2&1&1&0\\1&0&1&2&1\\-2&-1&-4&0&-1\end{pmatrix} \sim \begin{pmatrix}1&1&0&2&1\\0&0&1&-3&-2\\0&-1&1&0&0\\0&1&-4&4&1\end{pmatrix} \sim \begin{pmatrix}1&1&0&2&1\\0&1&-1&0&0\\0&0&1&-3&-2\\0&0&-3&4&1\end{pmatrix} \sim \begin{pmatrix}1&1&0&2&1\\0&1&-1&0&0\\0&0&1&-3&-2\\0&0&0&-5&-5\end{pmatrix} \sim \begin{pmatrix}1&1&0&2&1\\0&1&-1&0&0\\0&0&1&-3&-2\\0&0&0&1&1\end{pmatrix}$$

$\dim P = 3$, $\dim Q = 2$, $\underbrace{\dim(P+Q) = 4, \text{ báze } P+Q}_{40\%} = \left(\begin{pmatrix}1&2\\1&-2\end{pmatrix}, \begin{pmatrix}1&2\\0&-1\end{pmatrix}, \begin{pmatrix}0&1\\1&-4\end{pmatrix}, \begin{pmatrix}2&1\\2&0\end{pmatrix} \right)$

$\wedge\ P+Q \subset\subset \mathbb{C}^{2,2}$

$\Downarrow$

$P+Q = \mathbb{C}^{2,2}$

nebo třeba standardní báze $\mathbb{C}^{2,2}$ $\left(\begin{pmatrix}1&0\\0&0\end{pmatrix}, \begin{pmatrix}0&1\\0&0\end{pmatrix}, \begin{pmatrix}0&0\\1&0\end{pmatrix}, \begin{pmatrix}0&0\\0&1\end{pmatrix} \right)$

$\dim(P \cap Q) = \dim P + \dim Q - \dim(P+Q) = 1$

$\underbrace{\text{báze } P \cap Q = (\mathbb{X})}_{40\%}$

$\mathbb{X} = 1 \cdot \begin{pmatrix}2&1\\2&0\end{pmatrix} - 1 \cdot \begin{pmatrix}1&0\\1&-1\end{pmatrix} = \begin{pmatrix}1&1\\1&1\end{pmatrix} \in Q$

kontrola:

$-2\begin{pmatrix}1&2\\1&-2\end{pmatrix} + \begin{pmatrix}1&2\\0&-1\end{pmatrix} + \begin{pmatrix}0&1\\1&-4\end{pmatrix} = \begin{pmatrix}-1&-1\\-1&-1\end{pmatrix} \in P$

doplněk P do $\mathbb{C}^{2,2}$ např. $\left[\begin{pmatrix}2&1\\2&0\end{pmatrix}\right]_\lambda$

20%

③ (a) Pracujeme se souřadnicemi ve standardní bázi:

60%

$$\begin{pmatrix}1&1&1&3\\1&-1&1&1\\-1&2&0&0\\0&0&1&0\end{pmatrix}\sim\begin{pmatrix}1&1&1&3\\0&-2&0&-2\\0&3&1&3\\0&0&1&0\end{pmatrix}\sim\begin{pmatrix}1&1&1&3\\0&1&0&1\\0&0&1&0\\0&0&0&0\end{pmatrix}$$

$M=[x_1,x_2,x_3]_\lambda$

báze $M=(x_1,x_2,x_3)$

(b)

40%

$$\left(\begin{array}{ccc|c}1&1&1&0\\1&-1&1&0\\-1&2&0&1\\0&0&1&1\end{array}\right)\sim\left(\begin{array}{ccc|c}1&1&1&0\\0&-2&0&0\\0&3&1&1\\0&0&1&1\end{array}\right)\sim\left(\begin{array}{ccc|c}1&1&1&0\\0&1&0&0\\0&0&1&1\\0&0&0&0\end{array}\right)$$

$x\in M$, lze ho doplnit na bázi

↑ odsud je vidět, že (x_1,x_2,x) je báze M

④ $y=\left(\begin{pmatrix}0\\0\\-1\end{pmatrix},\begin{pmatrix}0\\1\\0\end{pmatrix},\begin{pmatrix}2\\0\\0\end{pmatrix}\right)$

(a) $\vec{x}=1\cdot\vec{y_1}-1\cdot\vec{y_2}+2\cdot\vec{y_3}=\begin{pmatrix}0\\0\\-1\end{pmatrix}-\begin{pmatrix}0\\1\\0\end{pmatrix}+2\begin{pmatrix}2\\0\\0\end{pmatrix}=\begin{pmatrix}4\\-1\\-1\end{pmatrix}$

45%

$\vec{y}=\vec{x_2}-3\vec{x_3}=\begin{pmatrix}0\\2\\1\end{pmatrix}-3\begin{pmatrix}0\\1\\1\end{pmatrix}=\begin{pmatrix}0\\-1\\-2\end{pmatrix}$

$\vec{z}=(\vec{y})_y=\begin{pmatrix}\alpha_1\\\alpha_2\\\alpha_3\end{pmatrix}=\begin{pmatrix}2\\-1\\0\end{pmatrix}$ $\quad\vec{y}=\alpha_1\vec{y_1}+\alpha_2\vec{y_2}+\alpha_3\vec{y_3}$

$\begin{pmatrix}0\\-1\\-2\end{pmatrix}=2\begin{pmatrix}0\\0\\-1\end{pmatrix}-1\cdot\begin{pmatrix}0\\1\\0\end{pmatrix}+0\cdot\begin{pmatrix}2\\0\\0\end{pmatrix}$

(b) Stačí ověřit LN, protože $\dim\mathbb{R}^3=3$.

25%

$$\begin{pmatrix}4&0&2\\-1&-1&-1\\-1&-2&0\end{pmatrix}\sim\begin{pmatrix}1&1&1\\0&-1&1\\0&-4&-2\end{pmatrix}\sim\begin{pmatrix}1&1&1\\0&1&-1\\0&0&1\end{pmatrix}$$

Ano, W je báze $\mathbb{R}^3$.

(c) $2\vec{x_1}-\vec{x_3}=\begin{pmatrix}2\\1\\1\end{pmatrix}$

30%

$$\left(\begin{array}{ccc|c}4&0&2&2\\-1&-1&-1&1\\-1&-2&0&1\end{array}\right)\sim\left(\begin{array}{ccc|c}1&1&1&-1\\0&-1&1&0\\0&-4&-2&6\end{array}\right)\sim\left(\begin{array}{ccc|c}1&1&1&-1\\0&1&-1&0\\0&0&-6&6\end{array}\right)$$

$\left(2\vec{x_1}-\vec{x_3}\right)_W=\begin{pmatrix}1\\-1\\-1\end{pmatrix}$

$\begin{pmatrix}2\\1\\1\end{pmatrix}=1\begin{pmatrix}4\\-1\\-1\end{pmatrix}-1\begin{pmatrix}0\\-1\\-2\end{pmatrix}-1\begin{pmatrix}2\\-1\\0\end{pmatrix}$

1. kontrolní test B

(1.) (a) $V \subset\subset \mathbb{C}^3$, tudíž V je vektor. prostor nad $\mathbb{C}$

70%

$V \neq \emptyset$ (např. $\begin{pmatrix}0\\0\\0\end{pmatrix} \in V$)

V je uzavřená na $\oplus$ a $\odot$:

$\forall \vec{x} = \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} \in V \quad \forall \vec{y} = \begin{pmatrix}y_1\\y_2\\y_3\end{pmatrix} \in V \quad \forall \alpha \in \mathbb{C} \qquad \vec{x} \oplus \vec{y} = \begin{pmatrix}x_1+y_1\\x_2+y_2\\x_3+y_3\end{pmatrix} \in V$

platí totiž $x_1 = 0 = x_2$, $y_1 = 0 = y_2$, tudíž

$x_1 + y_1 = 0 = x_2 + y_2$

$\alpha \odot \vec{x} = \begin{pmatrix}\alpha x_1\\\alpha x_2\\\alpha x_3\end{pmatrix} \in V$

platí totiž $x_1 = 0 = x_2$, tudíž $\alpha x_1 = 0 = \alpha x_2$

30%

(b) V není vektor. prostor nad $\mathbb{C}$

argumenty např. V není uzavř. na $\oplus$: $\underset{V}{\underset{\Uparrow}{\begin{pmatrix}1\\0\\0\end{pmatrix}}} \oplus \underset{V}{\underset{\Uparrow}{\begin{pmatrix}0\\1\\0\end{pmatrix}}} = \begin{pmatrix}1\\1\\0\end{pmatrix} \notin V$

(2.) Pracujeme se souřadnicemi ve standardní bázi.

$$\begin{pmatrix} 2 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 2 & 2 \\ 2 & 1 & 1 & 1 & 0 \\ 0 & -4 & -1 & -2 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & 2 & 2 \\ 0 & -2 & 1 & -3 & -3 \\ 0 & -1 & 1 & -3 & -4 \\ 0 & 0 & -5 & 10 & 15 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & 2 & 2 \\ 0 & 1 & -1 & 3 & 4 \\ 0 & 0 & -1 & 3 & 5 \\ 0 & 0 & 1 & -2 & -3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & 2 & 2 \\ 0 & 1 & -1 & 3 & 4 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix}$$

(columns 1–3: Q; columns 4–5: P)

$\dim Q = 3$, $\dim P = 2$, $\dim(P+Q) = 4$ } $P+Q = \mathbb{R}^{2,2}$
$P+Q \subset\subset \mathbb{R}^{2,2}$

báze$(P+Q) = \left(\begin{pmatrix}2&1\\2&0\end{pmatrix}, \begin{pmatrix}0&1\\1&-4\end{pmatrix}, \begin{pmatrix}1&0\\1&-1\end{pmatrix}, \begin{pmatrix}1&2\\1&-2\end{pmatrix}\right)$ nebo např. standard. báze $\mathbb{R}^{2,2}$ $\left(\begin{pmatrix}1&0\\0&0\end{pmatrix}, \begin{pmatrix}0&1\\0&0\end{pmatrix}, \begin{pmatrix}0&0\\1&0\end{pmatrix}, \begin{pmatrix}0&0\\0&1\end{pmatrix}\right)$

dim+ báze $P+Q$ = 40%

$\dim(P \cap Q) = \dim P + \dim Q - \dim(P+Q) = 1$

báze$(P \cap Q) = (X)$ $\qquad X = 2\begin{pmatrix}1&2\\1&-2\end{pmatrix} - 1\begin{pmatrix}1&2\\0&-1\end{pmatrix} = \begin{pmatrix}1&2\\2&-3\end{pmatrix} \in Q$

kontrola: $-1\begin{pmatrix}2&1\\2&0\end{pmatrix} - 1\begin{pmatrix}0&1\\1&-4\end{pmatrix} + 1\begin{pmatrix}1&0\\1&-1\end{pmatrix} = \underset{P}{\underset{\Uparrow}{\begin{pmatrix}-1&-2\\-2&3\end{pmatrix}}}$

dim + báze $P \cap Q$ = 40%

doplněk Q do $\mathbb{R}^{2,2}$ např. $\left[\begin{pmatrix}1&2\\1&-2\end{pmatrix}\right]_\lambda$

20%

③ (a) Pracujeme se souřadnicemi ve standardní bázi:

60%

$$\begin{pmatrix} 1 & 1 & 1 & 3 \\ 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ -1 & 2 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & -2 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 3 & 1 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$M = [x_1, x_2, x_3]_\lambda$

báze $M = (x_1, x_2, x_3)$

(b)

40%

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ -1 & 2 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -2 & 0 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -2 \end{array}\right)$$

$x \notin M$, nelze jej doplnit na bázi M

④ $\mathcal{Y} = \left(\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right)$

(a) $(\vec{x})_{\mathcal{Y}} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \Rightarrow \vec{x} = \vec{y_1} + \vec{y_2} - 2\vec{y_3} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} - 2\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$

45%

$$\vec{y} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} - 3\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix}$$

$$\vec{z} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} -5 \\ 1/2 \\ 0 \end{pmatrix} \qquad \vec{y} = \alpha_1 \vec{y_1} + \alpha_2 \vec{y_2} + \alpha_3 \vec{y_3}$$

$$\begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix} = -5\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + 0 \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

(b)

25%

$$\begin{pmatrix} 2 & 1 & -5 \\ -2 & 0 & 1/2 \\ -1 & 5 & 0 \end{pmatrix} \sim \begin{pmatrix} -1 & 5 & 0 \\ 0 & -10 & 1/2 \\ 0 & 11 & -5 \end{pmatrix} \sim \begin{pmatrix} -1 & 5 & 0 \\ 0 & 1 & -9/2 \\ 0 & 0 & 1 \end{pmatrix}$$

Stačí ověřit LN, protože $\dim \mathbb{R}^3 = 3$.

$\mathcal{U}$ je báze $\mathbb{R}^3$.

(c) $(\vec{y})_{\mathcal{U}} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, protože $\vec{y} = 0 \cdot \vec{x} + 1 \cdot \vec{y} + 0 \cdot \vec{z}$

30%

1. kontrolní test C

(1.) $P \neq \emptyset$ (např. $\begin{pmatrix}0\\0\\0\end{pmatrix} \in P$), P je uzavř. na operace

50%

$\forall \alpha \in \mathbb{C}$ $\forall \vec{x} = \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} \in P$ $\forall \vec{y} = \begin{pmatrix}y_1\\y_2\\y_3\end{pmatrix} \in P$

ANO, $P \subset\subset \mathbb{C}^3$

$$\alpha \vec{x} + \vec{y} = \begin{pmatrix}\alpha x_1 + y_1\\ \alpha x_2 + y_2\\ \alpha x_3 + y_3\end{pmatrix} \in P$$

platí totiž $x_1 - 2x_2 + x_3 = 0$, $y_1 - 2y_2 + y_3 = 0$, proto

$$(\alpha x_1 + y_1) - 2(\alpha x_2 + y_2) + (\alpha x_3 + y_3) =$$
$$= \alpha \underbrace{(x_1 - 2x_2 + x_3)}_{0} + \underbrace{(y_1 - 2y_2 + y_3)}_{0} = 0$$

- $\begin{pmatrix}1\\0\\-1\end{pmatrix}, \begin{pmatrix}2\\1\\0\end{pmatrix}$ jsou LN vektory z P $\Rightarrow \dim P \geq 2$
- $\begin{pmatrix}1\\0\\0\end{pmatrix} \notin P \Rightarrow P \neq \mathbb{C}^3 \Rightarrow \dim P \leq 2$

Závěr, $\dim P = 2$

50% a $\left(\begin{pmatrix}1\\0\\-1\end{pmatrix}, \begin{pmatrix}2\\1\\0\end{pmatrix}\right)$ je báze (stačí ověřit LN).

(2) $\begin{pmatrix}1 & 2 & 1\\ 1 & 1 & 0\\ 0 & -1 & -1\\ 0 & 0 & 1\end{pmatrix} \sim \begin{pmatrix}1 & 1 & 0\\ 0 & 1 & -1\\ 0 & 0 & 1\\ 0 & 0 & 0\end{pmatrix}$ $\dim P = 3$

10%

$$\begin{array}{cc} \overbrace{\quad\quad\quad}^{Q}\ \overbrace{\quad\quad\quad}^{P} & \\ \begin{pmatrix}1 & 0 & 0 & 1 & 2 & 1\\ 0 & 1 & 2 & 1 & 1 & 0\\ -1 & 0 & 0 & 0 & -1 & -1\\ 0 & -1 & 3 & 0 & 0 & 1\end{pmatrix} & \sim \begin{pmatrix}1 & 0 & 0 & 1 & 2 & 1\\ 0 & 1 & 2 & 1 & 1 & 0\\ 0 & 0 & 5 & 1 & 1 & 1\\ 0 & 0 & 0 & 1 & 1 & 0\end{pmatrix}\end{array}$$

$\dim Q = 3$ $\quad \dim(P+Q) = 4$, $P+Q \subset\subset \mathbb{R}^4$ $\Big\}$ $P+Q = \mathbb{R}^4$

10%

dim a báze $P+Q$ = 40%

$$\text{báze}(P+Q) = \left(\begin{pmatrix}1\\0\\-1\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\\-1\end{pmatrix}, \begin{pmatrix}0\\2\\0\\3\end{pmatrix}, \begin{pmatrix}1\\1\\0\\0\end{pmatrix}\right)$$

nebo např. standardní báze $\mathbb{R}^4$

dim a báze $P \cap Q$ = 40%

$$\dim(P \cap Q) = \dim P + \dim Q - \dim(P+Q) = 2$$

$$\text{báze}(P \cap Q) = \left(\begin{pmatrix}1\\0\\-1\\1\end{pmatrix}, 1 \cdot \begin{pmatrix}1\\1\\0\\0\end{pmatrix} - 1 \cdot \begin{pmatrix}2\\1\\-1\\0\end{pmatrix}\right) = \left(\begin{pmatrix}1\\0\\-1\\1\end{pmatrix}, \begin{pmatrix}-1\\0\\1\\0\end{pmatrix}\right)$$

(3.) Pracujeme se souřadnicemi ve standardní bázi P_4.

$$\begin{pmatrix}-1 & 1 & 1 & 2 & 2\\ -2 & 1 & 2 & 1 & -1\\ 1 & -1 & 1 & -2 & 2\\ 1 & -2 & -2 & 1 & -1\end{pmatrix} \sim \begin{pmatrix}-1 & 1 & 1 & 2 & 2\\ 0 & -1 & 0 & -3 & -5\\ 0 & 0 & 2 & 0 & 4\\ 0 & -1 & -1 & 3 & 1\end{pmatrix} \sim \begin{pmatrix}-1 & 1 & 1 & 2 & 2\\ 0 & 1 & 0 & 3 & 5\\ 0 & 0 & -1 & 6 & 6\\ 0 & 0 & 1 & 0 & 2\end{pmatrix} \sim \begin{pmatrix}-1 & 1 & 1 & 2 & 2\\ 0 & 1 & 0 & 3 & 5\\ 0 & 0 & 1 & 0 & 2\\ 0 & 0 & 0 & 6 & 8\end{pmatrix}$$

75% $V = [\underbrace{x_1, x_2, x_3, x_4}_{LN}]_\lambda$, $\mathcal{X} = (x_1, x_2, x_3, x_4)$ je báze V

$(x_4)_{\mathcal{X}} = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$, protože $x_4 = 0 \cdot x_1 + 0 \cdot x_2 + 0 \cdot x_3 + 1 \cdot x_4$

25%

4. (a) $\mathcal{Y}$ je báze $\iff (\mathbb{Y}_1)_{\mathcal{X}}, (\mathbb{Y}_2)_{\mathcal{X}}, (\mathbb{Y}_3)_{\mathcal{X}}, (\mathbb{Y}_4)_{\mathcal{X}}$ LN $\quad \dim \mathbb{R}^{2,2} = 4$ víme

30%

$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

ANO

(b) $(\mathbb{Y})_{\mathcal{X}} = (\mathbb{Y}_1)_{\mathcal{X}} - (\mathbb{Y}_2)_{\mathcal{X}} + 2(\mathbb{Y}_3)_{\mathcal{X}} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} + 2\begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 1 \\ 2 \end{pmatrix}$

30%

(c) $\mathbb{X}_3 = \alpha_1 \mathbb{Y}_1 + \alpha_2 \mathbb{Y}_2 + \alpha_3 \mathbb{Y}_3 + \alpha_4 \mathbb{Y}_4 \iff (\mathbb{X}_3)_{\mathcal{X}} = \alpha_1 (\mathbb{Y}_1)_{\mathcal{X}} + \alpha_2 (\mathbb{Y}_2)_{\mathcal{X}} + \alpha_3 (\mathbb{Y}_3)_{\mathcal{X}} + \alpha_4 (\mathbb{Y}_4)$

40%

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{array}\right)$$

$$(\mathbb{X}_3)_{\mathcal{Y}} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ -1 \end{pmatrix}$$

1. kontrolní test D

(1.) $P \neq \emptyset$ (např. $\begin{pmatrix}0\\0\\0\end{pmatrix} \in P$), P je uzavř. na operace

50%

$\forall \alpha \in \mathbb{C}\quad \forall \vec{x} = \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} \in P \quad \forall \vec{y} = \begin{pmatrix}y_1\\y_2\\y_3\end{pmatrix} \in P$

$$\alpha \vec{x} + \vec{y} = \begin{pmatrix}\alpha x_1 + y_1\\ \alpha x_2 + y_2\\ \alpha x_3 + y_3\end{pmatrix} \in P$$

platí totiž $x_1 - 2x_2 + 2x_3 = 0$, $y_1 - 2y_2 + 2y_3 = 0$ / proto

$$(\alpha x_1 + y_1) - 2(\alpha x_2 + y_2) + 2(\alpha x_3 + y_3) = \alpha\underbrace{(x_1 - 2x_2 + 2x_3)}_{0} + \underbrace{(y_1 - 2y_2 + 2y_3)}_{0} = 0$$

ANO, $P << \mathbb{C}^3$

- $\begin{pmatrix}2\\0\\-1\end{pmatrix}, \begin{pmatrix}2\\1\\0\end{pmatrix}$ jsou LN vektory z P $\Rightarrow \dim P \geq 2$
- $\begin{pmatrix}1\\0\\0\end{pmatrix} \notin P \Rightarrow P \neq \mathbb{C}^3 \Rightarrow \dim P \leq 2$

závěr, $\dim P = 2$

50% a $\left(\begin{pmatrix}2\\0\\-1\end{pmatrix}, \begin{pmatrix}2\\1\\0\end{pmatrix}\right)$ je báze P (stačilo ověřit LN)

(2.) $\begin{pmatrix}1&1&1\\1&0&0\\0&-1&-1\\0&0&1\end{pmatrix} \sim \begin{pmatrix}1&1&1\\0&1&1\\0&0&1\\0&0&0\end{pmatrix}$ $\dim P = 3$ 10%

$\begin{pmatrix}0&1&-1\\1&1&2\\0&-1&1\\-1&-1&3\end{pmatrix} \sim \begin{pmatrix}1&1&2\\0&1&-1\\0&0&1\\0&0&0\end{pmatrix}$ $\dim Q = 3$ 10%

$$\begin{pmatrix}1&1&1&-1&0&1\\1&0&0&2&1&1\\0&-1&-1&1&0&-1\\0&0&1&3&-1&-1\end{pmatrix} \sim \begin{pmatrix}1&0&0&2&1&1\\0&1&1&-3&-1&0\\0&0&0&-2&-1&-1\\0&0&1&3&-1&-1\end{pmatrix} \sim \begin{pmatrix}1&0&0&2&1&1\\0&1&1&-3&-1&0\\0&0&1&3&-1&-1\\0&0&0&2&1&1\end{pmatrix}$$

30%

$\left.\begin{array}{l}\dim(P+Q) = 4\\ P+Q << \mathbb{R}^4\end{array}\right\}$ $P+Q = \mathbb{R}^4$ 10% báze$(P+Q) = \left(\begin{pmatrix}1\\1\\0\\0\end{pmatrix}, \begin{pmatrix}1\\0\\-1\\0\end{pmatrix}, \begin{pmatrix}1\\0\\-1\\1\end{pmatrix}, \begin{pmatrix}-1\\2\\1\\3\end{pmatrix}\right)$

40%

nebo např. standardní báze $\mathbb{R}^4$

40%

$\dim(P \cap Q) = \dim P + \dim Q - \dim(P+Q) = 2$ 10%

báze$(P \cap Q) = (\vec{x}, \vec{y})$ 30%

$$\vec{x} = +1\begin{pmatrix}0\\1\\0\\-1\end{pmatrix} - \begin{pmatrix}1\\1\\-1\\-1\end{pmatrix} = \begin{pmatrix}-1\\0\\1\\0\end{pmatrix}$$

$$\vec{y} = 1\cdot\begin{pmatrix}-1\\2\\1\\3\end{pmatrix} - 2\cdot\begin{pmatrix}0\\1\\0\\-1\end{pmatrix} = \begin{pmatrix}-1\\0\\1\\5\end{pmatrix}$$

③ Pracujeme se souřadnicemi ve standardní bázi P_5.

$$\begin{pmatrix} 1 & 1 & 1 & 2 & 2 \\ -1 & 1 & 2 & 1 & -1 \\ 1 & -1 & 1 & -1 & 1 \\ 2 & -2 & -1 & 1 & -1 \\ 1 & -1 & 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 2 & 2 \\ 0 & 2 & 3 & 3 & 1 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 3 & -3 \\ 0 & 0 & 2 & 1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 2 & 2 \\ 0 & 2 & 3 & 3 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

75% $V = [\underbrace{x_1, x_2, x_3, x_4}_{LN}]_\lambda$ $\quad \mathcal{X} = (x_1, x_2, x_3, x_4)$ je báze V

$(x_3)_{\mathcal{X}} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$, protože $x_3 = 0 \cdot x_1 + 0 \cdot x_2 + 1 \cdot x_3 + 0 \cdot x_4$

25%

④ (a) $\mathcal{Y}$ je báze $\Leftrightarrow (y_1)_{\mathcal{X}}, (y_2)_{\mathcal{X}}, (y_3)_{\mathcal{X}}, (y_4)_{\mathcal{X}}$ LN víme $\dim \mathbb{R}^{2,2} = 4$

30%

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{ANO}$$

(b) $(X_2)_{\mathcal{Y}} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} \Leftrightarrow (X_2)_{\mathcal{X}} = \alpha_1 (y_1)_{\mathcal{X}} + \alpha_2 (y_2)_{\mathcal{X}} + \alpha_3 (y_3)_{\mathcal{X}} + \alpha_4 (y_4)_{\mathcal{X}}$

40%

$$\left(\begin{array}{cccc|c} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right)$$

$$(X_2)_{\mathcal{Y}} = \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}$$

(c) $(Y)_{\mathcal{X}} = (y_1)_{\mathcal{X}} - 2(y_3)_{\mathcal{X}} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} - 2\begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ -2 \\ -1 \end{pmatrix}$

30%