The 25th Student Conference
"Winter School on Mathematical Physics"

24 – 30 January 2016
Janské Lázně, Czech Republic

Book of Abstracts

Krzysztof Bardadyn

Poincaré method in linear perturbation theory

In my speech I told how simple eigenvalue of complex hermitian matrix behaves after perturbation. Using implict function theorem I proved if in some close neighbourhood of simple eigenvalue of matrix before perturbation, exist exactly one eigenvalue of matrix after perturbation. I presented recurrent method of approximation eigenvalue of perturbated matrix.

Adam Brus

Basics of Orthogonal Polynomials

The orthogonal polynomials appears in many regions of modern physics. The definition of orthogonal polynomials is introduced and some basic theorems about orthogonal polynomials systems are stated. The existence of such a polynomials is discussed. An example of Tchebichef polynomials of first kind is shown.

Marián Fecko

Surfaces which behave like vortex lines

First, we show how stationary flow of ideal (and barotropic) fluid may be treated as a particular instance of the theory of integral invariants a la Poincare. Then, how Cartan's refined version of the theory leads immediately to non-stationary flow. In this formalism we check validity of classical theorems by Kelvin (on circulation) and Helmholtz ("vortex lines are frozen into the fluid"). Finally, we learn that there is a statement in integral invariant theory itself, which generalizes the Helmholtz theorem (appropriate more-dimensional surfaces are frozen into the "fluid").

Daniel Gromada

Construction of realizations of Lie algebras

The problem of realization of a Lie algebra structure by vector fields is widely applicable in group analysis of differential equations. In the talk a method for constructing Lie algebra realizations proposed by A. A. Magazev, V. V. Mikheyev and I. V. Shirokov is presented. The realizations can be computed from the structure constants only by means of matrix multiplication, exponentiation and inverting. If a classification of subalgebras of a given Lie algebra is known, then this method allows us to classify realizations of the Lie algebra very easily.

Michal Jex

Toy Models with Singular Interctions in 1D

In this lecture we present rigorous definitions and spectral properties of two kinds of localized point interactions on a line-δ and δ' . We are interested in behaviour of discrete spectrum with respect to the distance between two interactions. For this case we present asymptotic behavior of the negative point spectrum.

Helena Kolešová

Neutrino oscillations and masses

The 2015 Nobel Prize in Physics was awarded to Takaaki Kajita and Arthur B. McDonald for the discovery of neutrino oscillations, which shows that neutrinos have mass. For this reason, I would like to speak about the history of neutrino physics and to derive the oscillation formula which was essential for the discovery mentioned above.

Jaroslav Kysela

Equilibration in Quantum Networks

Thermal equilibrium and its formation, thermalization, are key concepts studied by statistical physics. Since these macroscopic phenomena emerge as a consequence of the underlying microscopic structure of the physical system, they should be described by means of quantum theory. Such a description is nevertheless still not fully understood. In our talk we focus on the equilibration feature of thermalization and consider quantum networks as models of many-body systems. We take into account two classes of the network evolution and demonstrate that under very general conditions the state of such networks tends to equilibrium for sufficiently long time spans.

Martin Malachov

Permutation, Pauli and Hadamard matrices as modifications of chaotic behaviour in entanglement purification

Computation and communication based on quantum physics promises some new and very convenients possibilities. However, it is needed to handle some problems, especially unavoidable noisy influence of the environment. This decoherence damages also quantum entanglement. Special schemes proposed to repair this important source are called entanglement purification protocols. One of the protocol employes measurment-based state selection in a special way to induce chaotic behaviour. The aim of this presentation is to give an overview of possible modifications to this protocol. Special types of matrices are used to modify the protocol and a new approach is used to identify induced chaotic behaviour. Presented concept of equivalent dynamics allows us to investigate behaviour of hundreds of realisable operators simply from knowledge of four specially chosen operators.

Jiří Maryška

Gibbs-like asymptotic states in quantum markov processes

In the presentation, we discuss the asymptotic dynamics of a finite-dimensional open quantum systems which undergoes the evolution according to a iterative completely positive map which is called a quantum markov process. Assuming the existence of so-called faithful invariant state, we show that the asymptotic state can be written in a closed analytical form by solving so-called attractor equations. Then we discuss the properties of the space of attractors (i.e solutions of the attractor equations), from which we conclude that the asymptotic states of quantum markov process can be viewed as a generalization of Gibbs state.

Lenka Motlochová

Fundamental domain of extended affine Weyl group

The extended affine Weyl group is an infinite extension of the finite Weyl group by shifts in the co-weight lattice. We consider only the simple Lie algebra A_2. At first, we describe the dominant Weyl chamber of the Weyl group of A_2. Then we present a fundamental domain of the extended affine Weyl group as a specific subset of the dominant Weyl chamber.

Josef Navrátil

Reaction-diffusion systems with non-differentiable nonlinearities

The classical Crandall-Rabinowitz theorem is useful for studying local existence of bifurcating branches for elliptic boundary value problems. In this talk, more general form of this theorem was introduced, concretely for elliptic problems with small non-differentiable perturbation. Application to system of reaction-diffusion equations was shortly mentioned.

Petr Novotný

Concepts of stability, stability of rotating free rigid body

Different types of stability of an equilibria state of an autonomous dynamical system will be defined and compared. Basic theorem concerning the stability as well as the method using Lyapunov function are presented. For Hamiltonian systems so called Energy-Casimir method is described and applied on a rotating free rigid body.

Iveta Semorádová

Time-dependent metrics in crypto-Hermitian quantum mechanics

The concept of three Hilbert spaces as a guideline to fully understand crypto-Hermicity of quantum observable is presented. In this framework we generalize the condition of unitarity of the evolution operator to a metric-dependent quasi-unitarity condition. Time-dependent Hamiltonians are considered and the issue of time-dependent metric operators is further explained.

Josef Schmidt

Isaacson's high frequency limit

The framework for dealing with gravitation waves on curved background was presented. Decomposition of metric into slowly varying background and high frequency perturbation was introduced and formalized defining so called steady coordinates. Ricci and Riemann tensors have been expanded in powers of frequency/amplitude of perturbation leading to Einstein equations of leading and next-to-leading order. Gauge invariance of leading order terms was established as metric decomposition gives rise to gauge freedom via coordinate transformation.

Stanislav Skoupý

Quantum walks and their use in search algorithm and quantum communication

We introduce discrete time quantum walks and illustrate their properties in the case of the quantum walk on discrete line. We present and describe quantum walk search algorithm on hypercube. The algorithm can be modified and then can be used to communication across the graph.

Aneta Sliżewska

Introduction to an inverse semigroups theory

The aim of this talk is to introduce the definition of an inverse semigroup, some properties of the set of idempotents and partial isometries. There will be also given some examples crucial for the inverse semigroup theory, for instance, symmetric inverse semigroup, CAR inverse semigroup and Cuntz inverse semigroup.

Zuzanna Sztabińska

Kummer shape algebra

In my paper I present some results from the paper "Classical and quantum Kummer shape algebras" of A. Odzijewicz and E. Wawreniuk. I present a family of integrable systems of nonlinearly coupled harmonic oscillators. Presented method of integration is based on the reduction of the system to a Hamiltonian system on $\mathbb{R}^3$ with Poisson-Nambu bracket given by circularly symmetric function $\mathcal{C}$. In this way one obtains the trajectories of the reduced system as intersections of the 0-level surface $\mathcal{C}^{-1}(0)$ of $\mathcal{C}$, called Kummer shape, with the level sets of the reduced Hamiltonian.

Michal Širaň

Lie's Third Theorem for L_∞-algebroids

An integration of L_∞-algebroids (a simultaneous generalizations of both Lie algebroids and L_∞-algebras) will be presented; the principal application is the integration of Courant algebroids. Joint work with P. Ševera. (arXiv:1506.04898 [math.DG])

Mária Šubjaková

Gaussian optics from linear

In linear optics we study rays that pass through an optical system close to its optical axis. In order to describe the trajectories of such rays, there is a matrix method in which we characterize a ray in a reference plane perpendicular to the optical axis by a four-dimensional vector. The optical system is then characterized by a 4×4 matrix. In my talk I discuss the possible forms of these matrices as well as the forms of matrices describing optical systems which possess some kind of symmetry. Namely, optical systems with rotational symmetry about their optical axis, optical systems in Gaussian optics and optical systems with mirror symmetry with respect to the planes, in which their optical axis lies.

Daniel Vašata

Entropic uncertainty relations

Heisenberg's uncertainty principle represents a fundamental limit to the precision with which position and momentum of a quantum particle can be known simultaneously. The principle is given by an inequality setting a lower bound for the multiple of variances of position and momentum. Very interesting alternative to this formulations is given by the so called entropic uncertainty principle that is formulated as the lower bound for the sum of Shannon differential entropies of the momentum and position. In the talk we introduce entropic uncertainty relations, derive them from the so called Babenko-Beckner inequality, and show their relations to classical Heisenberg's uncertainty principle. In particular we show that it is generally stronger with a clear superiority in special cases which are discussed.

Elwira Wawreniuk

Quantum Kummer shape algebra

In this talk, in analogy to the classical case presented by Zuzanna Sztabińska, we consider a quantum system of nonlinearly coupled harmonic oscillators. As a consequence of the reduction procedure, we obtain an operator algebra generated by three operators. These operators satisfy the relations which depend on the structural function $\mathcal{G}_\hbar$ defined by the interaction part of the considered Hamiltonian. This algebra, called a quantum Kummer shape algebra, describes the symmetry of the considered quantum system and, in the limit $\hbar \to 0$, corresponds to the classical Kummer shape algebra. We show this correspondence using the $*_\hbar$–product defined by the coherent state map of the reduced system. We also show that this correspondence intertwines the classical and quantum reductions of the system.

Václav Zatloukal

Electrodynamics with geometric calculus

In this talk, I will rewrite four Maxwell equations as a single equation using the language of geometric (or Clifford) algebra and calculus. To this end, I will first discuss the geometric interpretation of the Dirac spacetime algebra, from which I will deduce, in a natural way, the Pauli algebra of space. Some simple solutions of the Maxwell equations will be provided in order to illustrate the calculational capabilities of the presented approach.

Martyna Żuk

Wagner representation theorem

The main point of this talk is the proof of the Wagner representation theorem. This theorem shows that any inverse semigroup S is isomorphic to an inverse subsemigroup of partial bijections of S. This theorem is an analogue to Cayley theorem of group representations.