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Book of Abstracts

Adam Brus
Killing algebra of PP-wave metric

We study the Killing algebra of plane parallel wave metric and application of such metric in the Poisson-Lie T-duality. We show interesting steps in solving of Killing equations and find out the complete Killing algebra of general plane parallel wave metric. We construct such algebras for special cases and look for their 4-dimensional subalgebras which acts freely and transitively on spacetime. Such subalgebras are needed for application of PLT-duality.

Iva Bezděková
Limit distributions for quantum walks

For large times, the velocity density of quantum walks can be calculated. This density allows us to easily find the position probability distribution of the walker or the moments. The density depends on the choice of the initial coin state. Usual expression using the standard coin basis can be therefore simplified by a convenient choice of the new basis. We show possible simplifications on two similar two-state quantum walks and comment the extension to higher-dimensional quantum walks.

Filip Hložek
Operators in the theory of metamaterial cloaking

We summarize some basic facts about metamaterials and possible applications of them. We introduce operators derived from Maxwell's equation and justify the complexification in the perturbed "invsibility" operator.

Michal Jex
Spectrum of delta' interaction supported by a curve

We introduce rigorous definition of delta' interaction in one and two dimension. We show the spectral properties for the situation of delta' interaction supported by closed and non-closed curves in $\mathbb{R}^2$. Unlike other attractive potentials in $\mathbb{R}^2$ delta' interaction can be without bound states for the sufficiently weak coupling on non-closed curves.

Jan Korbel
On the Lambert $W-$function

The Lambert $W-$function is defined to be the multivalued inverse of the function $x \cdot \exp(x)$. It has many applications in mathematics and physics, some of whichare the subject of the talk, as e.g. delay differential equations, solutions of trinomic equations, infinite towers, eigenproblem of double-delta potential in quantum mechanics or MaxEnt distributions of specific classes of entropies.

Jan Kotrbatý
Automatic identification of low-dimensional Lie algebras

We introduce possible ways of identification of Lie algebras. Especially, we discuss use of the so-called invariants for this purpose. We focus on a couple of concrete invariants, namely definition of certain series of ideals and the generalized Casimir invariants.

Martin Malachov
Chaos in quantum information

Present technology is based on computational devices using bits. Quantum version of a bit - qubit may be base for future information technology. For example qubits represented by entangled states may be used for quantum teleportation and in other quantum phenomena. Although preparation of entangled states is still one of a crucial problems, it might be solved by implementation of so called purification protocols, which improve entanglement of system. This presentation aims on summary of basic facts about quantum information and purification protocols. Emerging of chaos in quantum physics is also discussed. Accent is then put on chaotical behaviour of system of qubits subject to action of nonlinear operator corresponding to one particular purification protocol.

Antonella Marchesiello
Third-order superintegrable systems with potentials satisfying nonlinear equations

The conditions for superintegrable systems in two-dimensional Euclidean space admitting separation of variables in an orthogonal coordinate system and a functionally independent third-order integral are studied. It is shown that only systems that separate in subgroup type coordinates, Cartesian or polar, admit potentials that can be described in terms of nonlinear special functions. Systems separating in parabolic or elliptic coordinated are shown to have potentials with only non-movable singularities.

Lenka Motlochová
Multivariate gaussian cubature formulas arising in connection with root systems

The term cubature formula is more or less a synonym for numerical integration of functions of several variables. Several methods of numerical integration arise from properties of orthogonal polynomials. There are, for example, cubature formulas connected to special cases of Jacobi polynomials associated to root systems. We describe these polynomials and we show basic ideas of the connection between cubatures and root systems.

Petr Novotný
A remark on Noether's theorem

Noether's theorem is a fundamental theorem in theoretical physics, which connects symmetries and conservation laws. Its formulation in classical mechanics usually require invariance of Lagrange function or action with respect to one–parametric group of transformations. We present proof of most general formulation of the theorem which involves transformation of time and where invariance is required up to certain additive term.

Josef Navrátil

Pattern formation in reaction-diffusion systems with unilateral condition

The theory of the Turing patterns is one of the most known concepts in Mathematical biology. Since 1952, when Turing posted his famous article, the reaction-diffusion systems have been widely studied and many new approaches to this theory have been developed. In this lecture the reaction-diffusion systems with unilateral terms will be introduced, the existence and estimates for critical points will be discussed and the existence of bifurcation points and their relation to the critical points will be mentioned.

Ivo Petr

Index calculus and the discrete logarithm problem

The discrete logarithm problem (DLP) is one of the two most important mathematical problems in the field of cryptography. We show the Diffie-Hellman key-exchange procedure and stress its vulnerability if the DLP could be easily solved. We also present the index calculus method, a historical concept which forms the core of the function field sieve method, the best algorithm that solves DLP in multiplicative groups of finite fields. The complexity of the problem is discussed as well.

Martin Prokš

Applications of Information theory in time series analysis

Information theory is not only a subset of communication theory. It intersects physics (statistical mechanics), mathematics (probability theory) or computer science (algorithmic complexity). The basic ideas of information theory such as entropy, mutual information and entropy rate can be used in time series analysis to measure the information flow between two series. This approach allows us detect statistical dependencies of all types, namely, linear and nonlinear temporal correlations.

František Růžička

Solvable models in PT-symmetric quantum mechanics

A short introduction in PT-symmetric quantum mechanics is presented, with emphasis on solvable models (models admitting exact construction of a metric operator). Algorithms for metric operator construction are reviewed, and several solvable models studies recently by the author are discussed in detail.

Iveta Semorádová

Crypto-Hermitian interpretation of Klein-Gordon equation

In this talk alternative view on Klein-Gordon equation is presented. We demonstrate Klein-Gordon equation in Schrödinger form, Hamiltonian of which is non-self-adjoint. Solutions to common problems with probability interpretation and indefinite inner product of Klein-Gordon equation are proposed by means of crypto-Hermititan interpretation of quantum mechanics. We introduce and compute family of metric operators which define new positive definite inner product on space of solutions of Klein-Gordon equation. Our Hamiltonian is self-adjoint with respect to this new inner product, although several new problems emerge.

Václav Svoboda
Tsallis entropy and generalized Black-Scholes formula

Tsallis entropy gives rise to non-extensive statistical mechanics. Via MaxEnt principle it leads to $q-$Gaussian distributions. These distributions play important role in systems exhibiting properties like fractal behavior or long range memory effects. From mathematical point of view significance of these distributions lies in framework of $q-$calculus. Further we will use $q-$Brownian motion to generalize famous Black-Scholes option pricing formula.

Martin Štefaňák
Modelling excitation transport with Quantum Walk

We study the efficiency of excitation transport to the sink on a ring. The propagation of excitation through the ring is modelled by a coined quantum walk. For a two-state quantum walk the excitation is always fully transferred into the sink, i.e. the efficiency is one. However, three exist three-state quantum walks for which part of the wave-packet is trapped on the ring. In such a case the excitation transport is not fully efficient. Nevertheless, we show that for some three-state quantum walks the trapping can be eliminated by dynamical percolation of the ring and the transport efficiency can be enhanced.

Jan Vábek
Theory of high-harmonic generation: quantum description of single-atom response

We are going to talk about interaction of an intense laser with a gaseous medium. A radiation with integer multiple frequencies of the laser frequency (high-harmonics) is generated in that medium. The basic principles of the high-harmonics generation will be presented. We are going to focus to the quantum description of single-atom response in the strong field approximation. This description will be placed in the context of the high-harmonics generation model.

Kamila Vysoká
Radiation hardness testing

The ALICE experiment at the CERN LHC accelerator is going to upgrade its inner silicone tracker detector (ITS). Besides precise tracking this detector has a role to determine location of primary and secondary vertices of particles leaving the interaction point and to provide fast triggering signal. Region close to the beam pipe is, however, exposed to large radiation load. Consequently, there might be non-negligible radiation damage induced on the used electronics and other parts of the detector. Therefore, it is important to test the radiation hardness of all detector parts and to search for radiation tolerant technologies and materials.

Jan Vysoký
Atiyah-Lie algebroid for pedestrians

Atiyah sequence and induced Lie bracket form a structure naturally associated with every principal G-bundle. Ordinary infinite cylinder can be endowed by a structure of trivial U(1)-bundle, and it thus allows a construction of Atiyah-Lie algebroid. All objects required in the process are explicitly calculated in cylindrical coordinates. A final form of the Lie algebroid bracket written in local coordinates is presented.

Izabela Wawreniuk
The spectral theorem for self adjoint operators and orthogonal polynomials

In this lecture is given spectral theorem for bounded self-adjoint operators in two different forms. The main point is to show that the spectral theorem gives connection between theory of bounded self-adjoint operators with simple spectrum and theory of orthogonal polynomials.

Pavel Winternitz
Superintegrability in classical and quantum mechanics: I and II.

A review is given of the theory of finite dimensional Hamiltonian systems that have more integrals of motion than degrees of freedom. The best known and until recently only known superintegrable systems are the Kepler-Coulomb systems and the isotropic harmonic oscillator. Classical superintegrable systems are interesting because for them all finite trajectories are closed and the motion is periodic. For quantum superintegrable systems the energy levels demonstrate a degeneracy that is not due only to geometric symmetries but rather to "higher" or "dynamical" symmetries. Moreover a conjecture confirmed by all known examples exists stating that maximally superintegrable systems ($2N-1$ independent well defined integrals of motion) are exactly solvable. We show that infinite families of superintegrable systems exist with integrals that are polynomials of arbitrary order in the momenta or even convergent series in the momenta. The integrals of motion form interesting non-Abelian algebras. These can be viewed as infinite-dimensional Lie algebras, but much more fruitfully as finitely generated polynomial algebras. Superintegrable systems are finite-dimensional analogs of soliton equations, like the Korteweg-de Vries equation, or the Kadomtsev-Petviashvili one.

Václav Zatloukal
Geometry via Clifford Algebra

We make a brief introduction into the theory of Clifford algebras with an emphasis on their geometric significance. Namely, we show how the inner and outer product are defined in terms of the fundamental Clifford (or geometric) product and how these provide an efficient algebraic description of basic geometric operations, such as projection, rejection, reflection, and rotation.
References:
1) D. Hestenes, G. Sobczyk, Clifford Algebra to Geometric Calculus, Springer (1987)
2) C. Doran, A. Lasenby, Geometric Algebra for Physicists, Cambridge Univ. Press (2007)

Hana Zemanová
The interaction of electron with the electromagnetic wave complex propagating in plasma in the Earth's magnetic field

After brief introduction to lightning and linear waves in plasma, the talk concentrates on electron motion in electromagnetic field. This field consists partly of homogeneous fields: electric field from storm cloud and magnetic field from Earth's magnetic field. Another part of this field is composed by R wave, which is created in plasma along magnetic field lines. In most cases, the electron is accelerated to speed of light becoming so-called killing electron.

Joanna Zonenberg
Boundedness of solution of four-dimensional difference system

In this talk we consider four-dimensional nonlinear difference system with deviating argument. Firstly, the classification of nonoscillatory solutions of the considered system is presented. Next, boundedness of a nonoscillatory solution is shown. The presented results are illustrated by example.

Martyna Żuk
Gauge groupoid

In this talk the gauge groupoid is considered as a particular example of Lie groupoid. The main point of the presentation is the theorem, that every transitive Lie groupoid is isomorphic with some gauge groupoid of a principal bundle $P(B, \pi, G)$.