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Lectures

Marián Fecko
Killing tensors

On a Riemannian (or pseudo-Riemannian) manifold (M, g), Killing tensors are generalizations of much more familiar geometrical objects, Killing vectors, to k-times contravariant (or, equivalently, covariant) completely symmetric tensor fields obeying Killing tensor equations. In particular, Killing vectors are then rank 1 Killing tensors and the metric tensor itself turns out to be rank 2 Killing tensor. Killing tensors arise naturally whenever one tries to find conserved quantities, which are polynomials in momenta (or velocities), with respect to Hamiltonian flow, generated by Hamiltonian, which is a polynomial in momenta (or velocities) as well (in particular, with respect to the geodesic flow). As an example, in the Kepler problem (whose Hamiltonian is a 2-nd order polynomial in momenta) we get as many as four rank 2 Killing tensors, the first one associated with conservation of the square of angular momentum (conserved quantity quadratic in momenta), the next three associated with conservation of each component of Runge-Lenz vector (being order 2 polynomials in momenta). All these rank 2 Killing tensors are reducible, meaning that they can be constructed from Killing vectors.

Student presentations

Iva Bezděková
General localizing coin for the three-state quantum walk

There is a lot of ways how you can try to find some conditions for a general 3×3 matrix to be a quantum walk coin that leads to the localizing quantum walk, but most of them do not give any result or cannot be completely solved. As a localizing coin we mean such $U(3)$ matrix that the time evolution operator of the walk with this coin has one constant eigenvalue It results in one more peak in the probability distribution of the walk. I will present one approach, where we find conditions for a general 3×3 matrix ('coin'), which lead to the constant eigenvalue of the time evolution operator. The matrix we have used it not really coin, because it is not unitary. We can create the most general parametrised unitary matrix as a product of three matrices, where two of them are trivial. The third matrix is complicated, but we can replace it by well-known mixing matrix used in standard model. If we apply the conditions we found on the parametrised unitary matrix with mixing matrix we get quite simple solution and the problem of localizing coin for the three-state walk is fully solved.

Adam Brus
Coboundary Poisson-Lie groups

In this speech we introduce into theory of Poisson-Lie groups and theory of Lie-bialgebras. We show that Lie-algebras corresponding to Poisson-Lie groups have natural Lie-bialgebra structure and we canonically define Lie-bracket on dual space to Lie-algebra. We define the Coboundary Lie-bialgebras using Lie-algebra cohomology and show how to build Poisson-Lie structure on corresponding Lie-group by use of Skylanin formula.

Tomáš Bzdušek
Analytic continuation of imaginary frequencies data to real axis

In condensed matter physics (CMP), linear response functions to external perturbations (e.g. magnetization to external magnetic field) are retarded Green's functions. Kubo showed that they can be expressed as certain thermal averages of an *unperturbed* system. In cases when the unperturbed hamiltonian cannot be solved explicitly, causal Green's functions with time-ordering operator may be used, leading to the usual perturbative approach of Feynman diagrams. In CMP a different method proposed by Matsubara is widely used. The so called *temperature Green's functions* are studied on time interval $t \in [i\beta, -i\beta]$ with $\beta = 1/T$. Their usage leads to Feynman diagrammatics as well, though it is convenient due to drastic simplification of practical numerical calculations. It leads, however, to a new problem of how to turn the resulting "imaginary frequencies" data into physically meaningful values of retarded Green's function. In my talk I will reproduce proof by Baym and Mermin of uniqueness of this procedure. I will also briefly mention two most widely used methods of analytic continuation which are Maximum-entropy method and Padé approximation.

Zofia Grabowiecka
Universal C*-algebras – general construction and examples

The purpose of the presentation is to introduce general construction of universal C*-algebra $C^*(\mathcal{G}, \mathcal{R})$ generated by a set of generators $\mathcal{G}$ subject to relations $\mathcal{R}$. Some examples of universal C*-algebras generated by different relations will be presented. Moreover Coburns' theorem will be used to show that Toeplitz algebra is isomorphic to universal C*-algebra generated by isometries. Uniqueness property of pair $(\mathcal{G}, \mathcal{R})$ will be defined and Cuntz algebra will be used as an example of universal C*-algebra that always has uniqueness property.

Lenka Háková
Reflection-generated polytopes

In this talk we study orbits of the action of reflection groups on the Euclidean space $\mathbb{R}^n$. We consider both crystallographic groups (Weyl groups of semisimple Lie algebras) and non-crystallographic groups. Our approach is geometrical since the points of each orbit form a set of vertices of a n-dimensional polytope. We mention some of the applications of reflection groups orbits and we show several examples in $\mathbb{R}^2$ and $\mathbb{R}^3$.

Pavel Heller
Non-standard numeration systems

In the talk, we introduce the concept of non-standard numeration systems. We focus particularly on the possibility of representing complex numbers in the Penney system (base $i-1$, digits $0, 1$). It is an example of a system that has some interesting properties, yet still retains many analogies with the classical systems (such as decimal). After demonstrating how to represent and manipulate with numbers in the system, we conclude by stating a theorem describing the whole class of numeration systems with properties virtually identical to the one presented.

Antonín Hoskovec
Classical computational operations & their quantum counterparts

I have presented past results on quantum cloning from two articles from 1982 by Wootters, W. K. & Zurek, W. H. and Dieks, D. Together all the results I have presented are known as the No cloning theorem. Following this example I have also compared classical operation of deletion to how it could be performed on quantum computers and using argumentation from an article by Pati, A. K. and Braunstein, S. L. from the year 2000 I have presented the so called No cloning theorem. Both results are or significant importance for quantum computing science.

Robert Jankowski
The one-third law

"One-third law" of evolutionary dynamics states that strategy A fixates in a B population with selective advantage if the fitness of A is greater than that of B when A has a frequency less than 1/3. Namely, if the basin of attraction of strategy B is less than one-third then an A mutant overcomes its initial disadvantage and fixates in the population with selective advantage.

Michal Jex
Photogeneration of charge carriers in 1D polymer materials

In this talk the short introduction to the problematic of photogeneration of free charge carriers in organic materials was given. Talk was focused on π-conjugated polymer materials for which there is dominant one dimension. Such systems are represented by substituted polyacetylenes with bulky side groups. The motivation for studying such materials is that they can be used in semiconductor parts in electronics as substitutes for classical inorganic semiconductors. Two models of photogenerations were introduced-original model of Archipov and modified Archipov model. Original model was created for modeling of materials doped with electron acceptors. Modified model can be used when there is resonance coupling between exciton state and charge transfer state.

Dalibor Karásek
Rudiments of Cohomologies of Lie Algebras

Cohomologies of Lie algebras are useful tools for the classification and for an encoding of various conditions, for instance for central extensions, in algebraic structure. It is a generalisation of de Rham complex of differential forms on a manifold.
The aim of this talk is to recall elementary definitions and demonstrate usefulness of cohomologies on the fact that the first cohomology group for the adjoint representation corresponds to the outer derivations of a given Lie algebra.

Barbara Krzemień
The Cantor-Bernstein Theorem

The main aim of this talk is to state and prove following theorem: Let A and B be any sets. If $|A| \leq |B|$ and $|B| \leq |A|$ then sets A and B have the same cardinality.

Jan Korbel
Can quantum mechanics be formulated as a stochastic process?

The stochastic approach to the quantum mechanics, escpecially to time evolution has been studied since 60's. The pioneer of that field was E. Nelson, followed by others. The main aim of the method is to describe quantum-mechanical time evolution as a stochastic process. The possibility of that approach is discussed. The natural assumption is to restict ourselves to Markov processes, because of similar properties, like the form of Schrödinger equation vs Fokker-Planck equation, and others. In the lecture is showed on an example of a two-state system, and it is the main result of the presentation, that it is not possible to describe quantum mechanics within Markov stochastic processes. In the end, the theory of Non-Markovian processes is mentioned and some possible stochastic approaches are suggested.

Jan Mareš
Quantum walks on percolated graphs

This talk consists of three parts. First it introduces the notion of quantum walks focusing mainly on a quantum walk on a line. The second section gives a brief description of percolation. There is no rigorous theory or sofisticated results involved, because we only use percolated graphs as a variant of disorder in quantum walks. The main part of the talk deals with quantum walks on percolated grahs. First we concern walks on a percolated line using the concept of quantum tunelling and the approach using dynamic gaps. In the last section, we sumarize results on asymptotic dynamics of quantum walks on percolated finite graphs. The conclusion here is, that the quantum walk does not have to reach the fully mixed state and periodic cycles may occure.

Gabriela Malenová
On spectral gap of quantum graphs

Quantum graphs are defined as differential operators on metric graphs satisfying standard matching conditions in all the vertices. Then the second smallest eigenvalue of the Laplacian (for connected graphs) is called spectral gap. We investigate the behavior of the spectral gap while cropping/enlarging the graph and compare it to combinatorial discrete case.

J Maryška
Asymptotic evolution of random unitary operations

In the talk we present an analytical form of the asymptotic evolution open quantum system resulting from large number of iterations of random unitary operations (RUO). It is shown, that the asymptotic dynamic of these systems is governed by a diagonalizable superoperator, which typically acts nontrivially in a low dimensional attractor space. Then we show an example of a quantum network consisting of N qubits, which are coupled by certain control unitary two-qubit interactions. We present a way how to visualize given RUO by a directed graph and we analyze the special class of these graphs, so called base graphs.

Radek Novák
Bound states in non-Hermitean quantum waveguides

In this talk we describe quantum waveguides, one of the objects of interest of mesoscopic physics. These structures are large enough to be manufactured on one side, small enough for the quantum effects to take place on the other. Starting from the physical description of an electron inside this quantum wire we approach the mathematical model for finding the bound states of a Laplacian inside an infinite strip subjected to Robin-type boundary conditions step-by-step. Further on we consider a weak compactly supported non-Hermitian perturbation in the boundary conditions, implied by the imperfection in the boundary of the waveguide which can cause the particle inside to escape. We unitarily transform the perturbation in the boundary conditions away for the price of a weakly coupled potential added to the Laplacian. The resolvent of this new Hamiltonian can be decomposed into the orthonormal basis and using the Birman-Schwinger principle we obtain conditions for the existence of a unique bound state and its asymptotic expansion in the powers of the coupling parameter.

Ivo Petr
Constrained Quantization With or Without Tears

In order to quantize a theory with gauge symmetries using canonical techniques, one has to face the fact that the Legendre transform, used to pass from Lagrangian to Hamiltonian formalism, is not well defined. A standard procedure developed by Dirac introduces constraints on the phase space, calculates their Poisson brackets which may lead to other constraints etc., and the whole algorithm may seem rather cumbersome. Another way to tackle the problem, which is due to Faddeev and Popov, treats a Lagrangian linear in velocities as its starting point. A canonical one-form is identified, brought into the canonical Darboux (or Darboux-Weinstein) coordinates, and the Euler-Lagrange equations are shown to be the Hamilton equations of motion, thus identifying the correct Hamiltonian. These equations lead to constraints whenever the two-form derived from the canonical one-form is degenerate. However, instead of keeping the gauge degrees of freedom, Faddeev and Popov solve these constraints, to obtain the gauge fixed Lagrangian. We compare the two techniques, and discuss possible issues one can encounter at the classical level.

Filip Petrásek
BCH formula and transformation of group coordinates for six-dimensional Drinfeld doubles

In this talk the mathematical method that allows us to find the transformation of group coordinates for solvable Drinfeld doubles is presented. At first defining the Drinfeld double as a connected Lie group and considering the theorem on the decomposition of solvable connected Lie groups we introduce Drinfeld double group coordinates. In the second part we consider Baker-Campbell-Hausdorff (BCH) formula as the solution of the exponential equation linking Lie groups and associated Lie algebras. Supposing particular commutation relations for Lie algebra elements we derive commutation formulas for corresponding Lie group elements. Finally using these results we find the transformation of Drinfeld double group coordinates for the decomposition $(7_0|1)$ and the dual one $(1|7_0)$.

Barbora Planková
Homogeneous droplet nucleation

Droplet nucleation plays an important role in processes such as formation of secondary aerosols in the atmosphere, nucleation in steam turbines etc. Important property of the nucleation is the so-called nucleation rate. To compute this, an important property must be determined – the work of formation of the droplet. This can be done using the classical nucleation theory or the gradient theory. Although more physically based is the gradient theory, due to occurred temperature deviation, their difference was not so evident compared to the experimental data.

Václav Potoček
Composing lenses

We introduce the concept of paraxial beams in geometrical optics and elementary transformations acting on the linear space of paraxial beams, parametrized by their displacement and slope. We show that the transformation applied by any centered optical system composed of thin or thick lenses and mirrors can be completely described by a matrix composed of elementary unimodular transformations and thus unimodular itself. As a main result, we prove that this claim can be reversed, i.e., not only any unimodular matrix of an appropriate dimension can be decomposed into a product of elementary geometrical optics transforms, but this can be done with the restriction that the decomposition actually represents a finite composite lens system, respecting the order of transforms given by light propagation.

Josef Schmidt
Conserved quantities for cosmological perturbations

Procedure for constructing infinite dimensional family of conserved currents in Einstein gravity has been described. The cornerstone of the method was the variational principle of arbitrary metric theory involving second derivatives – in our case Einstein-Hilbert action has been used. Infinitesimal displacements generated by arbitrary vector field together with a few differential geometry tricks lead to explicit formulas for conserved quantities. General geometric definition of cosmological perturbations has been described. Using above mentioned formalism we've given example of calculation of energy of Schwarzschild spacetime immersed into Minkowski spacetime.

Dominik Šafránek
Continuous many world interpretation

The Everett's many world interpretation tells us when an observer take a look at the Schrödinger's cat, with a certain probability he enters the world where the cat is alive and with certain probability he enters the world where the cat is dead. But what happens when he is opening the box slowly? What does he see? Can he reenter the world where the cat is alive to the world where the cat is dead? We show that considering continuous unitary transformation between closed and open box lead us to the highly non-trivial case, where the usual Everett's interpretation does not hold and need to reconsidered.

Helena Šediváková
Introduction to SU(5) grand unified theory

In the standard model of particle interactions, electromagnetic, weak and strong interactions are based on three different gauge symmetries, hence three independent coupling constants occur in the model. However, the numerical values of these constants depend on energy involved in the interaction, and it was believed that the three values meet in one point at energies of about 10^{16} GeV. According to grand unified theories (GUTs), the three interactions are unified above this scale and only one coupling constant is involved. The simplest GUT assumes that this universal coupling comes from the SU(5) symmetry. In my talk, this theory will be briefly introduced, the (wrong) prediction for the Weinberg angle will be derived and the ways how to modify the theory to make it consistent with the measurements (which is the subject of my research) will be sketched.

Matej Škovran
Possible effect of shear stress on cosmological perturbations

After a brief introduction to the standard cosmology, the theory of cosmological perturbations is discussed as well. At the end a new mechanism is proposed which could modify cosmological perturbations so that primordial gravitational waves (tensor perturbations) would be more likely to be observed in the near future even in the particle physics-motivated models of inflation (i.e. inflation on an energy scale below Planck scale). The mechanism is based on the interaction of perturbations with an elastic continuum filling the universe.

Šimon Valko
Unitary k-designs

The concept of a unitary k-design can be introduced as a distribution on a set of unitary transformations of density matrices which simulates uniformly chosen unitary transformation. A basic scheme for perfectly secure quantum state encryption known as private quantum channel is a 1-design. 2-designs have state encryption and other applications as well. With restriction to 1 q-bit, it can be easily shown what is the necessary and sufficient condition for the private quantum channel to achieve optimality in terms of Shannon's entropy of the key and number of transformations. However, the optimal case requires a source of perfect randomness which isn't aviable, but it is possible to modify the probability distribution to achieve compatibility with some imperfect sources of randomness.

Daniel Vašata
The generalized center of mass

The usual definition of the center of mass (also called centroid or the centre of gravity) of a subset in $\mathbb{R}^d$ is defined only for sets with finite and positive d-dimensional Lebesgue measure. It can be viewed as an expectation of the random variable uniformly distributed over the set of interest. For the purposes of the random closed sets theory it is however necessary to have such mapping defined on the sets with zero Lebesgue measure.
Our aim is therefore to construct the generalized center of mass that is defined for arbitrary compact subsets of $\mathbb{R}^d$. We present a natural extension of the definition of the center of mass that gives the same result on sets with positive Lebesgue measure. Such definition has also nice and natural properties for lover dimensional subset of $\mathbb{R}^d$ such as lines or points and leads to usual definition of the center of mass for those classes of objects.

Jan Vysoký
Rudiments of Graded Geometry

Graded geometry is a recent mathematical concept developed to unify long-established usage of non-commuting variables in physics. Although precise definitions use difficult language of algebraic geometry, basic principles can be demonstrated using simple examples. Basic idea of the assignment of the graded algebra of smooth functions to open subsets of graded manifolds is presented. It is shown that it is a completion of the algebra of polynomials in non-commuting variables. Analogies of vector fields of ordinary differential geometry are defined as graded derivations of the corresponding graded algebra of functions. Possible applications in the unification of various geometrical objects are given.

Václav Zatloukal
Local time of path integral

We recall the standard path-integral representation of the equilibrium density matrix of a quantum particle in a generic external potential. For investigating behaviour near zero temperature, it is advantageous to introduce the so called local time of a stochastic process. An alternative representation in terms of the local time is derived from the functional integral which is normally used in the quantum field theory. The merit of this new representation is that it is capable of investigating the equilibrium statistical physics at both, large and low, temperatures.

Hana Zemanová
Formation of lightnings and runaway breakdown

In the first part of the speech, I describe different types of lightning strokes including three not well known types (the blue jets, sprites and elves). Then, I describe basic theory of lightning formation which comes from Heinz Kasemir and others. I explain terms like stepped leader stroke, main and return stroke. Another part of my speech is dedicated to runaway breakdown. This mechanism creating a discharge area comes from Alexandr Gurevich, Gennady Milikh and Robert Rousell-Doupre. Finally, I derive and solve unrelativistic equation of motion for an electron in the D layer of ionosphere. From its solution it follows that by the work of electric field of ionosphere the electron surmounts energetic losses caused by collisions with air molecules and can be accelerated to the relativistic speeds. Therefore, I confirmed the existence of the runaway breakdown.

Joanna Zonenberg
Potential Symmetries for Partial Differential Equation

Discusses the method of introducing the potential symmetries for partial differential equation. Given a definition of nonlocal symmetries and following shown example of potential symmetry Burgers equation in form $u_{xx} - uu_x - u_t = 0$, where u_{xx} means second order partial derivative with respect to variable x.

T Zynda