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Book of Abstracts

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Lectures

Marián Fecko

Conservation laws in Nambu mechanics

Nambu mechanics was proposed in 1973 as an alternative to Hamiltonian mechanics. In the first part of the lecture, we remind how differential geometry enables one to see the relation between symmetries and corresponding conserved quantities in Hamiltonian mechanics. Then, using Takhtajan's version of the least action principle, we discuss peculiar relation between symmetries and corresponding conserved quantities in Nambu mechanics.

Student presentations

Mateusz Aniserowics
Problem of false coin

We have N coins and one of them is false, which means either heavier of lighter than the true ones (but we do not know which). Using two-armed scales we have to find the false coin and determine if it is heavier or lighter, usinf the lowest possible number of measurements. We will present an algorithm to solve this problem and show that we need only M measurements, where $M \geq \log_3(2N + 3)$.

Iva Bezděková
Localization and spreading of quantum walks

The three-state Grover walk on a line shows the localization effect characterized by a non-vanishing probability of the particle to stay at the origin. We generalize the Grover walk to the one-parameter family of walks which preserves its localization nature. Finally we show that the original Grover walk is the fastest one from the one-parameter family of localizing quantum walks.

Jan Fuksa
Generating function for $SU(2)$ Clebsch-Gordon series

Generating functions provide an useful tool in the theory of groups and their representations. Its remarkable feature is that it solves infinitely many problems of a given type at the same time. Thus a generating function for Clebsch-Gordon series is equivalent to a table of decomposition of all direct products of all irreducible representation of the group.
The aim of this talk is to show a simple example on generating functions. Specifically I show the generating function for $SU(2)$ Clebsch-Gordon series and describe its derivation.

Pavel Hoc
Search for Higgs boson

After a brief presentation on theoretical bounds of the Higgs boson mass, I discussed problems of findig it. There are just four relevant processes for creating the Higgs boson on p-p colliders, but to cover wide mass range we need to combine data from many decay channels. At the end of the presentation results from ATLAS and CMS experiments were shown.

Antonín Hoskovec
Excitation Transfer in Perturbed Networks

The presentation began by introducing the listeners briefly to the topic of state transfer in quantum networks and then continued by defining the problem to be solved, the excitation transfer on networks in form of linear equidistant chains with bending. Positive results of numerical integration and optimization algorithms on networks with uniform protocols were then presented and explained.

Robert Jankowski
Time delay of replicator dynamics

Replicator dynamics provides a dynamical way of achieving Nash equilibria in populations. In the social-type model of replicator equation the mixed Nash equilibrium is asymptotically stable point of dynamics for small time delay.

Michal Jex
When point interactions meet Quantum graphs

In this talk the introduction to problems of the quantum graphs with delta couplings was given. Brief review of rigorous mathematical construction of point interactions on the line was shown. Generalisation of the point interaction to finite edged graphs was given in the way of self-adjoint extensions of closed symmetric operators with proper boundary conditions at the vertices. Also certain important properties of the ground state was mentioned focusing on the relation between the length of the edges and the ground state energy.

Dalibor Karárek
Category theory — the art of arrows

Canonical and natural isomorphisms are occurring throughout both mathematic and physics, but definitions of these terms are rather vague. This fact gave rise to the category theory, which was then able not only to capture those ideas, but also developed new understanding of structures hidden behind mathematics.
In this talk basic definition of category, functor, and natural transformation will be presented. Those definition will be demonstrated on examples from daily practice. In the end the fact that isomorphism between a finite dimensional vector space and its doubledual is indeed canonical will be presented.

Samuel Kováčik
Quantum mechanics in noncommutative space

During the talk, I have introduced an example to show, why the space we live in could be fuzzy (non-commutative) on some very small scale. Shortly, I have discussed some benefits of fuzzy space for quantum field theory. Then, I've developed representation of commutation relation of fuzzy space. In this representation, I have defined hydrogen atom problem. In addition, I've shown exact solution of Hydrogen atom problem (energy spectrum) and principle of uncerainity in non-commutative quantum mechanics. All results were in agreement with commutative results (differences vanish when we set non-commutativity parameter equal to zero).

Radek Novák
Birman-Schwinger principle

We introduce a simple technique from the field of functional analysis used in singular perturbation theory. This principle allows us to transform a differential equation into an integral one. In the second part of the lecture we use it for the perturbation theory on the threshold of the essential spectrum to investigate a weakly-coupled bound state for the Hamiltonian $-\Delta+\varepsilon V$. During the calculations we discover the influence of the singularity of the Green function of the free Hamiltonian on the existence of bound states. We conclude with the asymptotic formula for the eigenvalue of the bound state.

Ivo Petr
Constrained Hamiltonian systems

Starting from the Lagrangian framework, the systems are analyzed, for which the Hessian matrix of the Legendre transformation is singular. It is shown, that for such a theory a set of constraints on the phase space emerges. Dirac's approach to the constrained system is presented and demonstrated on the example of the electromagnetic field. The primary first class constraints are analyzed as the generating functions of gauge transformations.

Barbara Pietruczuk
One-sided invertibility of $B - \lambda I$ generated by a linear map

A model class of weight shift operators B genarated by a linear map is considered. A description of properties of $B - \lambda I$ for λ belonging to the spectrum is given. In particular, a necessary and sufficient condition for $B - \lambda I$ to be one-sided invertible is obtained.

Barbora Planková
Fluid simulations

Computer simulations of fluids play an important role in physics since in many cases an experiment is not available and, by improving the computer abilities, simulations are more and more easy to be done. Because of weak bonds of fluids (comparing to solid states), bonds between two molecules can be described using very simple potentials, the most used one is the Lennard–Jones potential. Setting the bonds between molecules, the time evolution of the system can be established. To do this, two methods are used: Molecular dynamic method (MD) and Monte Carlo method (MC). While the MD uses simple integrator (e.g. Verlet integrator) to deterministically describe the movement of the molecules, MC uses random number generator to pseudo-randomly push each molecule in each time step. Using the thermodynamic principles, variables such as the pressure, free energy, chemical potential and more can be "measured" by the simulation.

Václav Potoček
Stimulated Raman Adiabatic Passage and Deterministic Single-Photon Manipulation

Stimulated Raman Adiabatic Passage (STIRAP) is an interesting and important application of the adiabatic theorem in Quantum Mechanics. It allows a 3-level atom in a Λ-configuration to be deterministically driven between the two low-energy eigenstates, effectively omitting the unstable excited state. When the fields are treated quantum optically, this transition can be shown to be accompanied by a transfer of exactly one photon between optical modes tuned to the two transitions. This transfer is fully controlled, takes place with asymptotically unit fidelity, and does not involve Rabi oscillations.
In the first part of the talk, we introduce the STIRAP configuration and the essentials of the Jaynes-Cummings model of interaction between a quantum optical field mode and a discrete-level atom. In the second part, we find the eigenenergies of the STIRAP system with one classical and one quantum field and show that in adiabatic transition, one photon is added to / removed from the quantum field while the atom undergoes a transition between its two metastable states. Finally, we show how this phenomenon can be used for controlled single-photon production, non-destructive projective measurement of the vacuum state, and creation of nonclassical field states in an optical cavity.

Jan Smotlacha
Coherent states of deformed parafermionic oscillator

We investigate the coherent states of the process described by the Hamiltonian

$$\widehat{H} = (\widehat{a}_1)^2 \widehat{a}_2^\dagger + \left(\widehat{a}_1^\dagger\right)^2 \widehat{a}_2 = \widehat{A} + \widehat{A}^\dagger ,$$

which corresponds to the deformed parafermionic oscillator. We denote $\widehat{M}$ the integral of motion of this process. Its form is $\widehat{M} = \widehat{n}_1 + 2\widehat{n}_2$, where $\widehat{n}_1, \widehat{n}_2$ are operators corresponding to the numbers of the appropriate particles. We restrict our attention to the case when $\widehat{M}$ and $\widehat{n}_1$ have even eigenvalues.

Dominik Šafránek
Free-will experiment
(application of delayed choice quantum eraser experiment)

Non-locality in Quantum theory leads to some counter-intuitive phenomena. One can actuall use the nonlocality to construct a device based on the delayed choice quantum eraser experiment, which could determine whether the tested person has or has not the free will. In other words, the device could possibly predict what the person is going to do before the person decides what to do. However, this device could be used for sending the information back in time, what may not be possible. At the speech the Free Will Experiment is introduced and some explanations why such device may not work are offered.

Helena Šedováková
Repeated interaction quantum systems

The interaction of a reference system with a chain of identical independent quantum systems is considered and the long time behaviour of the reference system is studied. As an example, the model for so called "one atom maser" is introduced, where the interaction of a single mode of electromagnetic field in a cavity with a chain of two-level atoms is described by Jaynes-Cummings Hamiltonian. A steady state of the electromagnetic field will be found for the case when the distribution of the incoming atoms energy is the thermal one. The mathematical methods will be presented on the toy model of two-level reference system.

Jan Vysoký
Nambu-Poisson structures

Nambu mechanics was an interesting generalization of Hamiltonian mechanics introduced by Nambu in 1973. Since that, Nambu-Poisson structures amazingly appeared in various fields of modern mathematical physics. Original form of Nambu bracket is presented. Axiomatization of important properties of Nambu bracket leads to the definition of Nambu-Poisson manifold. Takhtajan's reformulation of Nambu-Poisson manifold into the language of multivector fields is shown. Interesting geometrical and algebraical properties of Nambu-Poisson tensors are presented.

Elwira Wawreniuk
Fundamental grupoid

An introduction to the groupoid and the category theory is given. Covering maps of topological spaces and covering morphisms of groupoid are considered. The theorem about equivalence between category of covering maps of topological space and category of covering morphisms of fundamental groupoid of this space is presented.

Joanna Zonenberg
Classical Lie symmetries and integrable equations

It is shown infinite symmetries of selected equations such as Liouvielle equation, generalized equation of the form $u_{xy} = f(u)$, where $f(u)$ is smooth function, linear heat equation and nonlinear Burgers equation.

Jakub Železný
Density Functional Theory and Kohn-Sham Equations

The structure of solid matter is described by the many-body Schrödinger equation. This equation is too complicated to solve directly as the number of particles is very large. One way how to solve this problem is using the so-called density functional theory. In this theory the system is not described by wavefunction, but by much simpler function - the wavefunction density. Hohneberg-Kohn theorem shows that this density uniquely determines the system and that one can find the ground state density and ground state energy by minimizing certain density functional. Kohn and Sham improved this theory by transforming the system into a non-interacting one and writing the density using the (non-physical) wavefunction of this system.