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Book of Abstracts

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Lectures

Tomasz Czyzycki
Algebraic criteria of linearization of second order PDE

We construct algebraic criteria of linearization of nonlinear second order PDEs with two independent variables. To this aim we consider five dimensional subalgebra of Lie algebra of symmetry of studied equations. Criteria of linearization are given in the form of conditions for dimensions and ranks of such subalgebras. Further we apply this method to equations with three independent variables.

Marián Fecko
Canonical structure of Nambu mechanics

Nambu mechanics was proposed in 1973 as an alternative to Hamiltonian mechanics. In the lecture, I concentrate on its geometrical formulation, both for time-independent and time-dependent Nambu hamiltonians. In particular, I discuss Takhtajan's version of the least action principle.

Aurel Gábris
Higher dimensional quantum walks using optical delay loops

Quantum walk of a particle in one spatial dimension can be realized with a single photon in a compact optical setup using the polarization degree of freedom. At the heart of this optical setup is a polarization dependent delay line, which is combined with passive elements transforming the polarization state and with a feedback loop. Quantum walk in higher spatial dimensions require an extension of the internal degree of freedom to higher dimensionality space, and also more complex arrangement of delay lines. A excellent candidate for making this extension is to involve the orbital angular momentum states of the photon. I give a brief introduction to the orbital angular momentum quantum states, and illustrate how these states could be used to realize quantum walk on a 2 dimensional square lattice.

Student presentations

Lucie Augustovičová
Shape resonances

Potential barriers in atomic interaction often lead to shape resonances in which atoms are held together by the barrier. Two models of spherically symmetric potential barrier of simplified shape are discussed. Quasi-stationary levels are located using a boundary condition method and corresponding wavefunction are found.

Martin Bacovský
Central dependencies of $U_q(sl(2))$

Structure of the center of the quantum algebra $U_q(sl(2))$ is analyzed in detail for both cases when deformation parameter q is/is not a root of unity. The algebraic dependency is explicitly written out.

Iva Bezděková
Three-state quantum walk on a line

This presentation feature reader into the quantum walk. It concentrate on detailed introduction into the problem. First part is focused on two-states quantum walk on a line and the Hadamard transform. At the end of the first part is time evolution equation derived. Second part is focused on three-state quantum walk on a line. Why is important do study this type of walk. What happens, when the time evolution operator has at least one eigenvalue constant, which is not possible for the two-state quantum walk.

Kamil Červenka
Trace dynamics as a procursor of quantum field theory

We revisit the conceptual foundations of quantum theory by noting that QFT could be seen as an emergent phenomenon arising from statistical thermodynamics of matrix models with global unitary invariance. We describe the basic ideas of trace dynamics. Then applying statistical methods with some assumptions and approximations we show a correspondence between trace dynamics and quantum field theory.

Veronika Gáliková
Non-commutative quantum theory

The lecture deals with the transition from "standard" quantum mechanics to the modified theory, where the discrete space is the playground. The motivation for this step is, for example, the aim to dispose of some diverging integrals occurring in quantum field theories when considering the continuous space. However, the aim of this lecture is just to show the basic modifications one has to take into consideration when solving Schroedinger equation.

Pavel Hoc
Higgs mechanism

After a brief introduction to the Electroweak Theory, I describe the Higgs mechanism and spontaneous symmetry breaking, which causes that three gauge bosons and fermions become massive, while photon remains massless. I also show that it leads to the existence of one neutral Higgs boson.

Antonín Hoskovec
Perfect state transfer

Quantum computers in complete analogy to classical computers need a mean of communication between different computers as well as between the components of the computer itself. One of the methods of transporting the information encoded on qubits is the passive quantum wire. The passive quantum wire I presented was a network consisting of n arbitrary quantum objects that behave as two-level systems and that are mutually coupled. The information transferred was an excitation present at a site labeled as one and the excitation was transferred to n-th node. The rest of the network had to be prepared in ground states. Considering the transfer from the first site to i-th site a permutation in the basis, where the basis states are the states where excitation is present on the j-th site, I have presented that there are infinitely many Hamiltonians that lead to PST (Perfect State Transfer). I used for that purpose properties of exponential of linear operators on a finite dimension as that is the time evolution in quantum mechanics. The next step in the research is finding couplings in existing quantum computers that lead to PST, for example on trapped ions.

Robert Jankowski
Characteristic functions – examples and applications

It is presented the definition of character function. It is shown how to calculate the expected values by the characteristic function. Finally it is proved the theorem which says that there is one-to-one correspondence between characteristic functions and probability distributions.

Michal Jex
Ground state and point interactions in one dimension

We discussed the ground state of the point interaction Hamiltonian in one dimension. The talk was especially focused on introducing a mathematically rigorous definition of this Hamiltonian as the self adjoint operator on appropriate Sobolev space with boundary condition at the point interaction sites. Second part of my talk demonstrated relations between the ground state and the distance between point interaction sites on the line.

László Kekczkés
Chaos in the dynamics of qubits

The talk consisted of three parts. First I gave a quick reminder of the basic notions of the chaotical dynamics in one complex variable. Then a way to construct a nonlinear transformation of a single qubit density matrix was given. In the last part I showed, that with certain parameterization the dynamics of qubits can exhibit chaotical properties using the given nonlinear transformation.

Bálint Kollár
Quantum walks on the triangular lattice

We analyze the role of dimensionality in the recurrence properties of coined discrete quantum walks through the three-state quantum walk on the triangular lattice. We show that in the presented model the recurrence dynamics of the walker is completely controlled by the coin operator. We classify the available coins from this point of view.

Jan Korbel
Lévy distribution and hungry sharks

The presentation shows the importance of central limit theorem (CLT) in statistics on the example of classical random walk. After that the limitations of CLT are discussed, and the generalization is shown. From those it is possible to define Lévy distributions with heavy tails property. Moreover, there are shown some applications of Levy distributions to financial methods and environmental system models.

Gabriela Malenová
Hardy inequalities in the twisted quantum waveguides

We consider the spectral problem for the Laplacian in an infinite planar strip with Dirichlet and Neumann boundary conditions. It is known, that a twisted combination of the boundary conditions leads to the existence of Hardy-type inequalities. These inequalities can be interpreted as a sort of uncertainty principle.

Josef Navrátil
Energy momentum tensor in Poisson-Lie T-duality

In this lecture it was shown, how left-invariant field transform by Poisson-Lie T-duality, computed energy momentum tensor for sigma model, and also mentioned, that energy momentum tensor is conserved by Poisson-Lie T-duality transformation. Also basic properties of sigma models and of special type of Lie group called Drinfeld double were defined.

Radek Novák
PT symmetric model with magnetic field

A simple PT-symmetric system with constant magnetic field was presented. Some properties of the model were discused and the difference between selfadjointness and PT-selfadjointness was shown. Different boundary conditions were imposed on this problem.

Barbara Pietruczuk
Waszewski topological principle

The theorem of Wazewski which allows us to study asymptotic behaviour of solutions of the second order differential equation is presented together with the idea of the proof.

Václav Potoček
Quantum Lévi walks

Random walks have found their applications in virtually every field of science, most notably in mathematics, physics, biology, economy and computer science. This has inspired the study of a quantum analogy of random walks, called quantum walks, and their potential applications in quantum algorithms. It has been found that the hitting and mixing properties of quantum walks beat those of classical random walks, resulting in a quadratic speed-up of many algorithmic tasks. On the other hand, allowing so-called Lévy flights, i.e. occasional large steps with a heavy-tailed probability distribution, in classical random walks similarly leads to a significant enhancement of search effectivity, without leaving the classical regime.
In this talk, we study the possible combination of these two approaches, i.e., introducing Lévy flights in quantum walks within the framework of unitary time evolution. We pose the problem in a rigorous way and find two principal approaches which both result in solutions satisfying all the expected properties. The obtained systems represent possible implementations of a quantum Lévy walk on a line. We show that the basic properties exhibit a completely new kind of asymptotic behaviour, different from both classical Lévy random walks and non-Lévy quantum walks.

Josef Schmidt
Nagel-Schrenckenberg model

Nagel-Schreckenberg model is a 1D cellular automata model of a vehicular traffic. Principles of the model have been explained and its behaviour has been presented through basic characteristics such as fundamental diagram and distribution of distances and velocities. The crucial role of deceleration probability in a creation of traffic jams has been shown.

Petr Siegl
Introduction to PT symmetry

We introduce basic concepts of PT-symmetric quantum mechanics: operators with antilinear symmetry, P-self adjoint operators and metric operators. We present results on spectra of PT-symmetric operators and their similarity to self adjoint ones. The general concepts are illustrated on example of PT symmetric square well.

Jan Smotlacha
Electronic Structure of Carbon Nanohorns in a Uniform Magnetic Field

Nanostructured carbon materials are the materials with a special geometrical structure of their molecules which we call carbon nanoparticles. This geometrical structure is accompanied by topological defects in hexagonal plane lattice called graphen. In most cases, these defects originate from the presence of the pentagons for the positive curvature and the heptagons for the negative curvature. There are known variously shaped carbon nanostructures. The most famous is the fulleren which has the structure of the soccer ball and can be approximated as a sphere. It is composed of 60 carbon atoms which create 20 hexagons and 12 pentagons. But many other structures exist, for example nanocones, nanotoroids, nanohorns etc. Let us focus on the nanohorns. They have interesting electronic properties which enable a lot of practical applications. These molecules have a cylindrical form with a part of a fulleren-like molecule as a cap. More complicated structures of nanohorns can arise when two cylinders of different diameters are connected by a region with pentagon-heptagon pairs. It seems that the best approximation for pentagonal and heptagonal areas is hyperboloid - positively curved for pentagons and negatively curved for heptagons. The electronic properties of these structures can be explored by solving Dirac equation at a curved surface. From its solution, we calculate the local density of states for different values of the magnetic field and Compaq results for different kinds of defects.

Helena Šediváková
The Life Without Frenet

While introducing the Frenet frame in three dimensions, the curve is assumed to be twice differentiable. Moreover, if we want to introduce functions determining the derivatives of the frame as the torsion, it is necessary for the curve to be C^3. However, there is the way how to frame a C^2 curve as can be found e.g. in the article of Richard L. Bishop: There is more than one way to frame a curve. In the talk some results of the article are presented, the relatively parallel frame is introduced and it's properties are mentioned. Using of such alternative frames is in our case motivated by studying the quantum waveguides – the thin tubes built along the spatial curves – under mild regularity conditions on the support curve.

Lukáš Tomek
The ant on a gramophone disc (a problem of sub-Riemannian geometry)

The lecture deals with a simple problem – The ant on a gramophone disc. The ant starts at certain position on the disc, which is motionless in the beginning. When the ant is moving, the disc rotates according to angular momentum conservation. The task for the ant is to make such shortest roundtrip so that after returning to the starting position the disc will be turned by given angle. The problem is handled in sub-Riemannian geometry, which is a generalization of Riemannian geometry. Shortest roundtrips correspond to certain solutions of the equations for sub-Riemannian geodesics of the problem. Numerical solutions of the equations are presented.

Jan Vysoký
Growing Poisson-Lie vegetables

Poisson manifolds are an interesting generalization of a structure well known from the classical mechanics — Poisson bracket. Poisson bracket on Poisson manifold and Poisson bivector are introduced. If a Poisson manifold is also a Lie group with its multiplication compatible with the Poisson bracket, it is called a Poisson-Lie group. Poisson bivectors of Poisson-Lie group are of special kind, called multiplicative tensor fields. Interesting properties of multiplicative tensor fields are shown. Poisson-Lie groups are in direct relation to Lie bialgebras, special Lie algebras with dual Lie algebra structure on their dual spaces. A way to construct Lie bialgebra from Poisson-Lie group and vice versa is presented.

Jakub Železný
Simple PT-symmetric model

In PT-symmetric quantum mechanics one interprets a non-self adjoint operators as self-adjoint ones in a different Hilbert space with scalar product modified using the so-called metric operator. We present a simple PT-symmetric model and derive a formula for its general metric operator. We then use this formula to find the special case of a metric operator, the so called C operator, which is often used in PT-symmetric mechanics.