The 26th Student Conference
"Winter School on Mathematical Physics"

22 – 28 January 2017
Janské Lázně, Czech Republic

Book of Abstracts

Krzysztof Bardadyn

Introduction to Cuntz algebras

In my lecture I recalled basic definitions and facts of general theory of C* algebras. Afterwards I constructed Toeplitz algebra and I formulated Coburn theorem. Subsequently I defined Cuntz algebra and I gave some example of this algebra. In the end I was saying about structure of Cuntz algebras and I formulated Cuntz theorem.

Marián Fecko

Electrodynamics and non-inertial frames

For a given observer, electromagnetic field splits into its electric and magnetic parts. Technically, this can be conveniently done using differential forms. We provide an introduction to this machinery in general context and, in particular, we treat rotating frame.

Marie Fialová

Direct Integral and Landau Hamiltonian

The aim of the presentation will be to introduce the term Direct integral and explain its meaning in spectral analysis. The case of a particle on a plane in a constant magnetic field in one direction which is perpendicular to the plane is considered. Hamiltonian of such a system with specific calibration of the magnetic field (Landau calibration) is called Landau Hamiltonian. The spectrum of Landau Hamiltonian will be found using the Direct integral.

Filip Garaj

Cotton tensor

Cotton tensor is a useful object in pseudo-Riemannian geometry. It possess properties resembling those of a well-known and thoroughly studied Weyl tensor, however, Cotton tensor is often overlooked even though it is closely tied to the conformal geometry of the manifold. Aim of the presentation is to highlight this connection of Cotton tensor to other fundamental objects in geometry as well as to summarize some of its other useful properties. The vital property of Weyl tensor is the fact that it is an obstruction (its vanishing is a necessary and sufficient condition) to local conformal flatness of a pseudo-Riemannian manifold in dimensions four and higher. The Cotton tensor is an obstruction in a similar sense on three-dimensional manifolds. Additionally, in three dimensions, there is a dual Cotton-York tensor, that this presentation shall contain as well.

Daniel Gromada

Classification of Lie algebra realizations

In the presentation an algorithm for classification of Lie algebra realizations by vector fields will be presented. It is known that local transitive realizations are in one-to-one correspondence with subalgebras of the Lie algebra. Very effective methods for explicit computation of such realizations are also available. We are going to show that a classification of all realizations, in some sense, can be obtained from classification of the transitive ones.

Helena Kolešová

Unbearable lightness of neutrinos

According to recent experimental data the neutrino masses are less than 1eV which is about million-times smaller than the mass of any other particle we know. It is speculated that some kind of physics beyond the Standard Model of particle physics can explain this smallness. Different mechanisms of this kind will be presented together with the implications of such models.

Jan Kotrbatý

Lie fields and representations of the Poincaré groups

Irreducible unitary representations of the Poincaré group $\mathcal{P}_4$ are of fundamental importance in quantum physics. The representations were classified by E. P. Wigner in 1939 and later on his result was broadened to much wider class of Lie groups by G.W. Mackey. Within the talk, an alternative method for construction of irreducible unitary representations is suggested and illustrated on the Poincaré groups $\mathcal{P}_2$, $\mathcal{P}_3$ and $\mathcal{P}_4$. The technique is motivated by the famous Gelfand-Kirillov conjecture, namely we make use of the relationship between the fields of fractions corresponding toWeyl algebras and universal enveloping algebras, respectively. Connection to Mackey theory is discussed in each case in order to show that both methods lead to the same results.

Martin Malachov

Numerical Analysis of Chaos in Entanglement Purification of Mixed States

Successfull experimental realisation of entanglement purification would allow quantum communication to be performed effectively. One of proposed purification protocol exhibits chaotic features when acting on a special set of pure states. The aim of this lecture is to mention results gained from numerical analysis of the protocol acting generally on mixed states. Found results are discussed from the point of view of the reliability of numerical estimates and the practical applications.

Peter Mészáros

CMB anisotropies in the presence of elastic matter

Cosmic microwave background (CMB) anisotropies are given by radiation energy density perturbations at the time of recombination and effect influencing photons during their way to the observer. Equations governing evolution of cosmological perturbations, effects influencing photons after last scattering and computation of CMB angular power spectrum will be explained. In the last part I will talk about the effect of presence of solid matter on CMB anisotropies.

Lenka Motlochová

Application of Burnside's lemma in discrete Fourier-like analysis

Burnside's lemma is useful for mathematical problems involving symmetries. In the context of discrete Fourier-like analysis involving simple Lie algebras, it is used to determine the cardinality of a set of Weyl-orbit functions forming an orthogonal basis of functions sampled on a finite grid. The grid is given as a scaled finite fragment of a Weyl group invariant lattice, the dual root lattice, contained in a fundamental domain of affine Weyl group. In this talk, we derive an explicit formula for the number of points in the grid connected to the simple Lie algebra A_n.

Josef Navrátil

Critical and bifurcation points for equations with Lipschitz nonlinearities

The bifurcation for equations with smooth nonlinearities is described by celebrated Crandall-Rabinowitz Theorem and Krasnoselskii Theorem. One of the essential assumption is that the equation is linearizable. However, such assumption is no more true in systems with obstacles and more general in systems with Lipschitz continuous nonlinearities, where the natural assumption the existence of homogenization of the system. The most recent results for such systems will be presented, and also suggested main goal for future research. The main application of this results will be the reaction-diffusion equations.

Radek Novák

Grushin problem

We introduce a simple algebraic tool useful in the study of spectral problems. It arises as a generalisation of Schur's complement formula for matrices and allows us to reduce the studied problem to a simpler one described by an effective Hamiltonian. Its application is demonstrated on several simple examples.

Ivo Petr

Lattice-based cryptography

Whereas such problems as factorization of composite numbers or discrete logarithm problem can be solved effectively on a hypothetical quantum computer, no efficient algorithms are known for shortest/closest vector problem defined on a lattice of sufficiently large dimension. Therefore, lattice-based cryptography offers several candidates for post-quantum cryptography. In our talk we define lattices, shortest/closest vector problem in a lattice and present several cryptosystems, such as GGH or NTRU, whose security relies on the intractability of the underlying lattice problems. We also sketch the LLL algorithm, which is the most successful algorithm used to reduce the lattice basis and solve the shortest vector problem.

Václav Potoček

Orbital Angular Momentum of Light

I will introduce the orbital angular momentum (OAM) of light, a degree of freedom often overlooked compared to spin angular momentum (or polarization). I will provide an overview of the theoretical as well as experimental methods of studying OAM and discuss some of its practical uses. In the second half of my talk I will introduce a mathematical theory of optical vortex lines in beams carrying OAM as well as more general optical fields.

Bernard Rybołowicz

Continuous Curves

I introduce some different definition of continuous curves and some facts about space filling curves. I present examples such as first Peano curve, its formula and Hilberts geometric construction of curves like Wada lakes and Sierpinski Carpet.

Stanislav Skoupý

State transfer by means of discrete-time quantum walks

State transfer by means of discrete-time quantum walks We introduce the scheme of discrete-time quantum walk algorithm for the state transfer on an example of star graph. We consider the quantum walk search algorithm with two marked outer vertices, sender and receiver. We show how to simplify the problem by using square of evolution operator and by nding the invariant subspace with respect to the square of the evolution operator. We calculate the number of steps of the walk and we prove that the perfect state transfer is achieved. We brie y mention the results for other types of highly symmetric graph.

Tereza Štefková

Trapping in quantum walks

Quantum walks represent a quantum-mechanical counterpart of classical random walks. In this lecture we introduce the concept of discrete-time quantum walks and illustrate its basic characteristics on the example of quantum walks on a line and on a two-dimensional lattice with a special focus on the effect of trapping.

Vojtěch Teska

Orthogonal Subalgebras and Finite Quantum Systems

The algebra $M_n(\mathbb{C})$ is used to describe finite quantum systems. The presentation will include overview of the structure of fine gradings of $M_n(\mathbb{C})$ and its relation to maximal abelian groups of commuting *-automorphisms (MAD-groups), Pauli's group of "almost commuting" matrices and quasi-orthogonal sublagebras of $M_n(\mathbb{C})$ which are connected to the unsolved problem of classification of mutually unbiased bases in $\mathbb{C}^n$.

Matěj Tušek

On some extensions of the Iwatsuka model

After a short introduction to the two-dimensional magnetic Laplacians we will focus on the case when the magnetic field is invariant with respect to the translations in one direction. We will present several sufficient conditions for the absolute continuity of the corresponding operator. We will also shortly discuss a generalization to the three-dimensional quantum layers.

Patrik Urban

Introduction to conformal (Weyl) theory of gravity

The aim of the presentation will be to introduce the basic ideas of conformal theories and problems associated with their use. Specifically the theory of Weyl gravity will be presented with its fourth-order equations of motion. The analysis of Pais-Uhlenbeck oscillator will serve as an example to show, that the problem of higher order theories might be solvable by finding proper form of the Hamiltonian.

Jan Vábek

Full Saddle Point Approximation For Single Active Electron Response During High Harmonic Generation

The presentation will focus on a microscopic response describing the interaction of an intense IR-pulse with an atomic target leading to the generation of high harmonics of the fundamental field. In the talk, there will be introduced a solution for a single active electron response using the so-called Saddle Point Approximation technique. This solution will be adiabatically extended for pulses where a slowly varying envelope can be used. In the second part, the intrinsic phase of the generated field will be recognised from the solution and there will be shown possible ways leading to controlling spectral properties of the generated field and its possible applications.

Daniel Vašata

Duality between the spectra of Schrödinger operators on quantum graphs and discrete Laplacians on underlying combinatorial graphs.

We consider magnetic Schrödinger operators on quantum graphs with equilateral edges. Using the technique of boundary triples and Krein's resolvent formula, the spectral problem is reduced to the discrete magnetic Laplacian on the underlying combinatorial graph. In particular, it is shown that the spectrum on the quantum graph is the pre-image of the combinatorial spectrum under a certain entire function.

Kateřina Zahradová

How to frame a curve

In this talk we discuss three different ways how to construct a moving frame along a curve in three dimensions and also generalize two of those concepts to higher dimensions. We also study the advantages and disadvantages of different choices of the frames. Many practical applications of framing illustrate the uses of these frames in different areas of our lives.

Václav Zatloukal

Generalizing Complex-Plane Integration Formulas to Higher Dimensions

Popular results of complex integration theory, such as the Cauchy integral formula or the Residue theorem, are generalized to a setting where the complex plane is replaced by a Euclidean space of arbitrary dimension. This is achieved by employing the Fundamental integration theorem of geometric calculus, and appropriately generalizing the notion of holomorphic functions.