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Book of Abstracts

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P. Baxant
Spontaneous Parametric Down-Conversion as a Source of Quantum Entangled Photons

The construction of quantum computer based on physical representation of qubit by photon requires successful realisation of the generation of quantum entangled photons. Spontaneous parametric downconversion (SPDC) is one of the auspicious way to achieve this goal. In this speech we will introduce some basic facts about SPDC and its quantum description. Then we will focus on SPDC in array of nonlinear waveguides and discuss the spatio-spectral correlations of the output states.

I. Bezděková
Stability of Point Spectrum for Three-state Quantum Walks on a Line

Evolution operators of certain quantum walks posses, apart from the continuous part, also point spectrum. The existence of eigenvalues and the corresponding stationary states lead to partial trapping of the walker in the vicinity of the origin. This feature was found in the three-state walk on a line with the Grover coin operator, where the evolution operator has one eigenvalue equal to unity. Similarly, Grover walk on a square lattice also has a point spectrum. We analyze the stability of this feature for three-state quantum walks on a line subject to homogenous coin deformations. We find two classes of coin operators that preserve the point spectrum. These new classes of coins are generalization of coins found previously by different methods.

A. Brus
***p*-Simplicial Geometry and Incidence Matrices**

In this talk we define p-simplex and boundary operator and its properties. Then we introduce the Chain group and its subgroups (Cycle and Boundary group.) These groups gives us informations about topological properties. First we show how these groups looks like for 2 examples (simple triangle and then for simplest triangulation of Möbius strip). We define the Homology group as group of nonbounding Cycles. At the end we define the Incidence matrix, show how it look like for our examples and show what usefull informations it provides.

T. Czyżycki
Linear Filtering, DCT and their Applications

We present the problem of linear filtering and discrete cosine transforms for finite sequences of data. We show spectral densities, transmitation functions and their properties for important linear filters. Further we focus on discrete cosine transforms and discuss their eight kinds related to different boundary conditions of the signal. We show the possibilities of generalization of DCT in multidimensional case.

Z. Grabowiecka
Elements of Adjoint Order 2 of Simple Lie Groups

The purpose of this presentation is to list the elements of adjoint order 2 of a compact simple Lie groups $\mathfrak{G}$ of rank n. $\mathbb{Z}_2$ grading of Lie algebra $\mathfrak{g}$ by an automorphism of order 2 is presented. Root system of Lie algebra $\mathfrak{g}$ is explained and properties of $\mathfrak{g}$ are discussed in terms of extended Dynkin diagrams and Cartan matrices. With the use of fact that each element $g \in \mathfrak{G}$ of finite order n is given by non-negative and relatively prime integers $[s_0, s_1, \ldots, s_n]$ and the marks of $\mathfrak{G}$, the formula for adjoint order of element $g \in \mathfrak{G}$ is presented. The elements of adjoint order 2 are listed for all kinds of Lie algebras.

L. Háková
Weyl Group Orbit Functions in Image Processing

Several families of multidimensional special functions, called orbit functions, are introduced in the context of the Weyl groups of semisimple Lie algebras. These functions are discretely ortogonal when summed up over a finite lattice grid in $\mathbb{R}^n$. This allows us to provide a Fourier-like transforms of functions defined on such grid. In this talk we will speak about applications of two-dimensional orbit functions in image processing via so-called orbit convolution.

R. Jankowski
The Maximum in the Fixed Point

A function from the theory of information involving probability density function is presented. It has a unique property: Its maximal value lies in the fixed point of this function. This attribute is very usefull in finding parameters of various distributions.

M. Jex
Spectrum of Strong δ' Interaction Supported by a Closed Loop

In the talk we give rigorous definition of generalized Schrödinger operator in $L^2(\mathbb{R}^2)$ describing an attractive singular interactions. We calculate the spectrum for δ' type supported by a $C4$-smooth closed curve Γ of length L without self-intersections. For strong coupling limit, i.e., the coupling parameter approaches zero $\beta \to 0_+$, it is shown that the number of eigenvalues can be written as $\frac{2L}{\pi\beta} + \mathcal{O}(|\ln\beta|)$. Furthermore we are able to write the j-th eigenvalue in the same limit as $-\frac{4}{\beta 2} + \mu_j + \mathcal{O}(\beta|\ln\beta|)$. In the previous expression μ_j is the j-th eigenvalue of the Schrödinger operator on $L2(0, L)$ with periodic boundary conditions and the potential $-\frac{1}{4}\gamma 2$ where γ denotes the signed curvature of Γ.

M. Juránek
Sign Homomorphisms of Weyl Groups

In the first part, the notion of Weyl groups, their fundamental domain and their classification using Dynkin diagrams is reminded. The heart of the presentation is the definition of sign homomorphisms and how to find them using the compatibility relation. With this relation, we show that there are only four viable sign homomorphisms. In the last part, even subgroups are defined using sign homomorphisms. Lastly, the fundamental domain of even subgroups is constructed using the fundamental domain of Weyl groups.

D. Karásek
Geometric Formulation of Time Dependent Hamiltonian Mechanics

Hamiltonian mechanics has revolutionized our way of dealing with physical problems. Not only that it reveals new methods of solving physical systems, but it is the base point for the quantum mechanics. The connection between functions and transformations of a phase space is really astonishing and the approach to the integrals of motions is very beautiful.
It is not surprising that many of these qualities come from geometric properties. However to extend these properties to the time dependent Hamiltonian is not a straightforward task. We will offer a method of dealing with Hamiltonian dynamic with help of vortex lines and Poincaré-Cartan invariants.

J. Kysela
Twirling Operations in Quantum Computation

Recent research on quantum computation has led to the development of several important classes of quantum algorithms with unprecedented capabilites, such as reliable cryptographic methods or efficient factoring techniques. In some of these algorithms, e.g. entanglement purification protocols, one special kind of quantum operations is employed. These so-called twirling operations transform arbitrary state of a quantum system into a Werner state which is easier to deal with in further processing. We provide a definition of twirling operations and demonstrate some of its important properties. Finally, the iterative method for computation of one particular case of twirling is presented and possible limitations of this method are discussed.

J. Lochman
Dimensional Reduction

The existence of more than four dimensions in nature, even if they were small, may not be completely harmless. The aim of this lecture is to present a concept called dimensional reduction, which is able to reveal some visible manifestations of extra dimensions in our four dimensional world. At the beginning it will be shown, that effects of quantum gravity could be observed at LHC, if teories of extra dimensions are correct. This will be done by derivation of fundamental mass scale. As the second example we will assume scalar field living in 5 dimensions and using dimensional reduction we reveal, how does this field appears to observer in four dimensional world.

O. Löw
Quantum Walks – The Moment Method

Random walks are used intensively in algorithms designing. Using the quantum effects, the so called quantum walks are promising formalism, which can be used to implement quantum algorithms and quantum systems simulations. In the first part of the lecture we introduce the formalism of quantum walks. The notation and wave approach being established we describe a very useful method of studding quantum walks asymptotical behavior – the moment method.

M. Malachov
Various Representations of Rotations and their Application in Crystallography

Rotation as an operator on a three-dimensional Euclidean space is well known and important object of many mathematical and physical branches of science. It is usually represented by a unitary matrix but sometimes it is much more useful to use another representation. Many of these will be descried. For example, representation using Euler angles is also very common thanks to its illustrative character and simple implementation. On the other hand there is an axes convention, which is hidden in the definiton of Euler angles and can cause trouble when not treated carefully. Some advantages and disadvantages of various, almost exotical representations are also to be presented together with a brief introduction into crystallography and its orientation description. Finally, tilt and twist representation will be accented as an instrument for an easy description of structural properties of processed magnesium.

A. Marchesiello
Superintegrability: Separability vs Higher Order Integrals

Although 2D superintegrable systems of second order have been well studied and completely classified, the same can not be said for higher order systems. In this talk I will present the main issues concerning 3rd order superintegrable systems and show how assuming separability in suitable coordinates systems might be a good strategy to solve some of them.

J. Maryška
Asymptotic Properties of Size Scalling Qubit Networks

In talk we present a simple collision model of system of qubits. In this model, so-called interaction graph is defined. This graph contains all information needed for describing of the asymptotic dynamic of given system. Then we modify the model by defining a random interaction graph and study how the asymptotic dynamic depends on the size of the qubit system. As a result we show that there exists a phase transition for certain size of the system which describes the information about the asymptotic state.

J. Navrátil
Reaction-diffusion Equations and Pattern Formation

The lecture will be about Turing idea of instability in reaction-diffusion equations, published in 1952. Turing supposed existence of some substances called morphogenes, which react and diffuse. This is described by a system of two nonlinear PDE's. There is one parameter in each equation - a diffusion constant. For some values of these parameters, the solution can be interpreted as a pattern. We will make a short introduction into reaction-diffusion systems and mention the situations in which patterns can arise. In some situations there can be a source in the system, described by so called unilateral condition. This leads to variational inequalities, which we will also mention. At the end we will sketch some ideas about these inequalities and future plans to research.

R. Novák
Influence of Dimensions in Quantum Mechanics

In this talk we show how the dimension of an underlying euclidean space can affect the properties of an self-adjoint operators acting in this space. In particular, we study the influence of a small potential perturbation on the spectrum of a free Laplacian depending on the dimensionality of the space, in which this operator "lives". We find out that in dimensions 3 and higher this perturbation cannot cause an emergence of the bound state (i.e., isolated eigenvalue of finite multiplicity) from the spectral threshold due to the Hardy inequality, whereas in lower dimensions opposite happens. These results are demonstrated on a well-known example of a finite potential well.

Petr Novotný
Graded Contractions of Lie Algebras and their Representations

The classification of all representations of a given solvable Lie algebra presents an open problem even in the case of three dimensional Euclidean Lie algebra e(2). The concept of graded contractions for representations of Lie algebras is presented. It allows one to construct representations of some solvable Lie algebras from known irreducible representations of simple Lie algebras. Criteria for faithfulness, indecomposability and nonequivalence of contracted representations are formulated in the terms of contraction matrices.

U. Ostaszewska
On Application of Gambling Team Technique to Waiting Time Problems in i.i.d. Sequences

Let (ξ_n) be a sequence of i.i.d. random variables valued in a finite alphabet Ω. Consider reduced set of patterns $\mathcal{A} = \{A_1, A_2, \ldots, A_k\}$ that is none of the patterns contains any other as a subpattern. Let assume that the pattern B, containing none of $A_1, A_2, \ldots, A_k$, is observed in the first m trials. We are interested in a random variable $\tau_{\mathcal{A}B}$ of the first time that one observes the pattern from $\mathcal{A}$ in the sequence (ξ_n) after one observed the initial pattern B. Applying the gambling team technique, introduced by Li (1980) and developed by Pozdnyakov and other authors, we are able to find formulas for the expected value and generating function of the waiting time $\tau_{\mathcal{A}B}$.

I. Petr
Plane Waves in General Relativity and String Theory

Spacetime metrics allowing nowhere vanishing covariantly constant null vector field play important role in the study of gravitational waves. We shall analyze a particular family of these metrics called plane waves, and show that the equations of motion of a point particle, as well as of a string propagating in such a background, are linear in the lightcone gauge and in principle can be solved. Due to this property, the plane waves constitute a playground suitable to study string solutions, especially in the presence of singularities of these spacetimes.

J. Prokop
Calculating Invariants of Lie Algebras by Method of Moving Frames

Generalized Casimir invariants of Lie algebras are of importance in numerous cases. They are useful in representation theory; they allow us to find physically significant quantities, the simplest example being the total angular momentum obtained as a quadratic Casimir operator in algebra $\mathfrak{so}(3)$, etc. Method of Moving Frames is an effective algorithmic way of computing these invariants. The aim of the talk is to present the method of Moving Frames. Firstly, the generalized Casimir invariants will be introduced; next the method outlined and applied to a simple example.

J. Schmidt
Lie Derivative of Nontensorial Objects

he Lie derivative is a well-known geometrical tool for measuring the change of tensors along given vector field. In this talk we are going to generalise this concept for geometrical objects on manifolds defined by its components and its transformation law. The diffeomorphism induced coordinates are used to define pullback of the above-mentioned objects. The notion of Lie derivative follows in a natural way. Computation of explicit formulas is done for two important examples – tensor densities and Christoffel symbols.

L. Strmisková
Thermodynamics in Hydrogen Fuel Cells

The principle of hydrogen fuel cell will be explained. The theoretical value of voltage generated by fuel cell will be calculated and compared with voltage generated by working fuel cell. The causes of the differences between theoretical value and experimental values will be discussed. Then the transport of water and protons inside fuel cell membrane will be described using linear non-equilibrium thermodynamics. This approach gives us set of three equations, that describe the fluxes of heat, protons and water inside the membrane. These equations can be rewritten in order to gain the differential equations for the gradient of temperature, chemical potential of water and electrical potential inside the membrane. These equations are easily numerically solvable, so their solution can be sketched.

K. Šramková
What You Can Do in Physics, When You Are Not a Theorist

For a proper study of a quark gluon plasma it is very important to measure as many particles as possible. Different decays may have different properties; For example, they can achieve higher transversal momentum. The reconstruction of the decay of positively charged kaon into tree charged pions is presented. Data used in this analysis are from the experiment STAR at accelerator RHIC, from run 11 at energy $\sqrt{s} = 39\text{GeV}$ in gold-gold collisions. In this reconstruction it is possible to go for transverse momentum up to 3 GeV/c. The particle reconstructed in this way can be used in the study of an elliptic flow or of a nuclear modification factor.

M. Tušek
Huygens' Principle in N Dimensions

Huygens' principle describes the propagation of a wave of light but it also applies to other wave-like phenomena. In its generalized form it states that any locus of constant phase propagates in the (N-dimensional) space at the same speed. However this is true if and only if N is odd. We will demonstrate it by a direct analysis of the general solution to the wave equation without sources. To keep the analysis simple we restrict ourselves to the cases $N = 1, 2, 3$.

D. Vašata
Generalized Definition of Centroid for Certain Subclasses of Compact Sets with Zero Volume

The centre of mass represents an important point of an physical object. When treating the dynamics of rigid body we often use coordinates that are connected with the centre of mass. Besides the importance in physics the centre of mass can be treated in a pure mathematical way. We will therefore present the definition of centroid for compact subsets of Euclidean space $\mathbb{R}^d$ with positive volume (d-dimensional Lebesgue measure) in the exact parallel to the physical definition of the centre of mass for objects with positive volume and constant density. Then we present the generalized version of the definition that works for some sets with zero volume. A sufficient condition of existence based on the measure geometric properties of sets in $\mathbb{R}^d$ will be presented. Finally, we show that the centroid exists for all rectifiable subsets with finite Hausdorff measure and particularly for compact convex sets.

J. Vysoký
Inverting of Singular Matrices: A Counterpart of Nambu-Poisson Structures?

It is known that invertible Poisson tensors correspond to symplectic forms. Nambu bracket of $p+1$ smooth functions is a natural generalization of the Poisson bracket of two functions. We seek for the differential form corresponding to Nambu-Poisson tensor. This procedure involves an inverse of the rectangular matrix. However; on Riemannian manifold one can use the orthogonal complements to fix the one-sided inverse uniquely. We examine the global properties of the corresponding differential form.

E. Wawreniuk
Reproducing Measure for the Basic Hypergeometric Series

The purpose of this presentation is to introduce the problem of finding the reproducing measure $d\mu_R = \frac{1}{2\pi} d\varphi d\nu_R$ when the reproducing kernel is given by the basic hypergeometric series, which was solved by A. Odzijewicz. It is shown that this problem is equivalent to the moment problem for the measure $d\nu_R$. The equation for the complex series related to the measure $d\nu_R$ is obtained. Solving this equation is equivalent to finding a reproducing measure $d\mu_R$. The formula for the solution of this equation is given.

H. Zemanová
Electron Motion in Plasma with External Magnetic Field

In the talk I describe a relativistic motion of an electron in a storm cloud, which is represented by homogeneous electric field. Stepped leader channel in a storm cloud is represented as a part of plasma in Earth's magnetic field, which is represented by homogeneous magnetic field. Studied electron interacts with R wave, which is created in plasma along magnetic field lines. In certain circumstances electrons can be accelerated to speed of light becoming so-called killing electrons.

J. Zonenberg

Oscillatory Properties of Solutions of the Fourth Order Difference Equations with Quasidifferences

A class of fourth order nonlinear difference equations with quasidifferences and deviating arguments was considered. It was presented the considered equation as a system of four-dimensional difference system. The sufficient conditions under which the considered equation has no quickly oscillatory solutions was given. Finally, the sufficient conditions under which the equation is almost oscilatory was presented.