19th Student Conference
"Winter School on Mathematical Physics"

Book of abstracts

24–30 January, 2010
Horní Polubný, Czech Republic

Lectures

A. Gabris
Quantumness witnesses

J. Hrivnák
Reflection groups and orbit functions

H. Hernandez
Geometric phases and quantum waveguides

Jan Kříž
Quantum waveguides — overview

We present an overview of commonly used methods and results on the spectral properties of planar quantum waveguides. We investigate the Laplace operator in a (possibly curved) planar strip subject to three different types of boundary conditions – Dirichlet, Neumann and a combination of these, respectively.

H. Lavička
Employment, Production and Consumption Model: Patterns of Phase Transitions

We have simulated the model of Employment, Production and Consumption (EPC) using Monte Carlo. The EPC model is an agent-based model that mimics very basic rules of industrial economy. From perspective of physics, the nature of the interactions in the EPC model represents multi-agent interactions where the relations among agents follow the key laws for circulation of capital and money. Monte Carlo simulations of the stochastic model reveal phase transition in the model economy. The two phases are the phase with full unemployment and the phase with nearly full employment. The economy switches between these two states suddenly as a reaction to a slight variation in the exogenous parameter, thus the system exhibits strong non-linear behavior as a response to the change of the exogenous parameters.

Student presentations

D. Dombek
Non-standard numeration systems I

The comtribution is devoted to positional numeration systems with non-standard base. We start by recalling some trivial properties of numeration with an integer base which is commonly used, e.g., with base equal to 10 or 2. Then we show how these properties change if we allow the base to be irrational – in accordance with Rényi definition of the so-called β-expansions. We continue by introducing quite recent generalisation made by Ito and Sadahiro, the $(-\beta)$-expansions, which use a negative base $(-\beta) < -1$. We recall known criteria for the admissibility of digit strings and then we define $\mathbb{Z}_{-\beta}$ – the set of so-called $(-\beta)$-integers. We describe a pathological case, when the only $(-\beta)$-integer is zero and we describe for which β such a situation occurs. Then we give a formula for describing distances between consecutive elements of $\mathbb{Z}_{-\beta}$ for a large class of β. In the end we define another possible generalisation, the so-called balanced $(-\beta)$-expansions. We describe some of their elementary properties and give the criteria for $(-\beta)$-admissibility in this numeration system.

T. Hejda
Substitutions fixing non-degenerate 3iet words

We describe morphisms, fixed points of which are words coding the dynamical system given by the exchange of three intervals with the permutation $(3, 2, 1)$, the so-called 3iet words. If the fixed point has factor complexity $\mathcal{C}(n) = 2n + 1$, the morphism can be assigned to a pair of amicable Sturmian morphisms. We as well show that all 3iet words come from pairs of amicable Sturmian words.

Dalibor Karásek
Classification of outer derivatives of Lie algebras

Subalgebras invariant with respect to automorphism (the so called megaideals) are very important in the clasification of Lie algebras, especially when constructing solvable extension of a given nilpotent algebra. We will introduce basic definitions and present another usage of megaideals and "nearly" invariant subalgebras.

K. Klouda
An algorithm for computing the least trimmed squares estimate

The least squares method is one of the most classical statistical tools. Unfortunately, the estimate given by this method was shown to be very sensitive with respect to

corrupted or contaminated data. In order to overcome this drawback, some robust methods have been proposed. One of the oldest ones is the least trimmed squares (LTS) method. Despite being very old, there is still known no satisfactory way how to compute the estimate given by the LTS method - especially in the case of larger data sets. In this talk a new exact algorithm for computing this estimate is introduced.

L. Motlochová
2D Generalization of symmetric and antisymmetric sine functions

Properties of the 2-dimensional generalizations of sine functions, that are symmetric or antisymmetric with respect to permutation of their two variables, will be described. It will be shown that the functions are orthogonal when integrated over a finite region of the real Euclidean space, and that they are discretely orthogonal when summed up over a lattice of any density. Analogues of the common cosine transforms I.,II.,III., IV. will be presented.

M. Mysliwiec
Magri method for the restricted trace – class operators

I. Petr
Homology group and an integral in $\mathbb{C}^2$

As the introduction, basic notions of the residue theory of one-dimensional complex analysis are summarized. Multidimensional analogue is introduced then using the theory of homology groups of a differentiable manifold. As an example, a two dimensional integral of a meromorphic function is evaluated.

B. Planková
Mathematical modeling of spherical interfaces

To the naked eye the interface between liquid and its vapor could seem infinitesimally thin. However, the truth is more complicated. In a layer just few nanometers thin the density of liquid drops to the density of vapor continuously. Description of this interesting behaviour in case of spherical droplets was a main subject of this talk. From simple thermodynamical principles and variation theory we derived second order ordinary differential equation and described, how to solve it for one component fluid (n-nonane) using van der Waals equation of state. Problems caused by this equation character and present possible way of fixing them were also mentioned. In the future we plan to use more sophisticated equation of state and to consider more complicated problems (two component fluid).

Václav Potoček
Computing with Quaternions

The aim of the lecture is to present an introduction to the quaternion algebra and its practical uses. First, basic definitions and lemmas will be presented, emphasizing the similarities and differences between quaternions and complex numbers. Several important representations of quaternions in computer data structures will be shown to simplify the actual computing with quaternions. As a straightforward application of the theory, the method of representing spatial rotations by unit quaternions will be presented. Finally, quaternions will be used to prove several important facts about the $SU(2)$ group in a very accessible way.

E. Pryzmont
The homotopy of topological groups related to Hilbert space

S. Siemien
Operations and effects

L. Strmisková
Shadowing theorem

M. Šarbort
Representation of motion in central potential by non-Euclidian surfaces

H. Šedováková
Effective Hamiltonian in curved quantum waveguides

The Dirichlet Laplacian in a curved two-dimensional strip built along a plane curve is investigated in the limit when the uniform cross-section of the strip diminishes. We show that the Laplacian converges in a strong resolvent sense to the well known one-dimensional Schroedinger operator whose potential is expressed solely in terms of the curvature of the reference curve. In comparison with previous results we allow curves which are unbounded and whose curvature is not differentiable.

V. Štěpán
Introduction to Supermanifolds

Introduction to A. Rogers' construction of supermanifolds, i.e., manifolds modelled over the infite-dimensional Banach superalgebra with superdifferentiable structure, i.e., structure being differentiable in the sense of Fréchet but, in addition, restricted by an extra condition.

T. Vávra
Non-standard numeration systems II

We study arithmetics in Ito-Sadahiro numeration system. For beta quadratic Pisot numbers we show that the set of numbers with finite expansion, $\mathrm{Fin}(-\beta)$, is closed under addition and that $\mathrm{Fin}(-\beta)$ is a ring, if β has positive conjugate. For quadratic Pisot units we determine the number of fractional digits that might appear when adding or multiplying two $(-\beta)$-integers.

J. Vysoký
Living in the superworld

J. Wasiliew
Toeplitz algebras

Václav Zatloukal
Supersymmetric Quantum Mechanics

Supersymmetric quantum mechanics (SUSY QM) was derived in 1980's by Edward Witten as a toy model for testing SUSY quantum field theories. However, it was soon adopted as a useful tool for solving quantum mechanical spectral and scattering problems. In this talk I present the basic ideas of SUSY QM – factorization of a Hamiltonian and definition of its, so called, supersymmetric partner. I show how to use these concepts to easily find the eigenstates of a certain type of Hamiltonians.